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# Endogenous leverage and expected stock returns

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This note clarifies conditions under which endogenous choice of debt induces a negative relation between leverage or default risk and expected stock returns. In the context of the model of George and Hwang [2009. Journal of Financial Economics 96, 56–79], we correct the contention that variation in bankruptcy costs across firms is sufficient. Variation in asset risk parameters can lead to the desired relation, but may not when also controlling for variation in book-to-market ratios. A simple parameterization of cross-sectional heterogeneity in risk and profitability implies a negative association of expected return with leverage and distress risk and a positive association with book-to-market.



## 1. Introduction

Does the endogeneity of firm debt explain the negative cross-sectional relation observed in the data between stock returns and leverage or default risk? A recent paper by George and Hwang (2009) asserts that capital structure optimization by firms which differ in their expected bankruptcy costs may yield such an association. If true, the hypothesis would nicely account for some puzzling and much debated findings that seem to fly in the face of the intuition that higher debt should amplify systematic risk exposures and so entail higher expected equity returns. More generally, their work speaks to an important emerging theme in asset pricing: namely, that cross-sectional heterogeneity in business characteristics may account for some of the observed anomalies in the cross-section of stock returns.

The model presented in their paper, however, does not support the proposed explanation for the leverage puzzle. Although their main proposition establishes that optimal debt does fall and expected returns do rise as costs of distress increase, for reasonable parameters the effect is very small. More

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importantly, the expected returns in the proposition are *firm* returns (i.e. debt plus equity) not stock returns. For equity, in fact, the model delivers the opposite result: as distress costs increase and optimal debt declines, expected stock returns *fall*. Since the empirical evidence pertains to (levered) stock returns, the puzzles are not consistent with the model's predictions.

This note shows, however, that the desired relationships can be restored by modifying the assumed heterogeneity. We show that, instead of differences in bankruptcy costs, differences in business risk are sufficient. Reasonable variation in risk can lead to strongly significant negative association between book debt and *equity* risk premia. We thus verify the thrust of the conclusion of George and Hwang (2009), that, in a world with diverse firms and endogenous levels of debt, the empirical findings may not be a puzzle.[1]

That is not the end of the story, though, because the empirical work in this area sets the bar higher than merely delivering univariate relationships. In particular, it is much harder to reproduce the results of regressions of returns on debt variables that also control for the ratio of equity book value to market value. The book-to-market ratio summarizes the same risk characteristics captured by leverage. Further complicating things, distress risk and (book) debt levels may be negatively related in the model. Thus accounting for a negative return-distress risk finding is not merely a corollary of a negative return-book debt relationship. In multiple regression, distress risk proxies may remain significantly negative in the presence of book debt.

Accounting for the desired multivariate associations requires heterogeneity along multiple dimensions. We show that the model can meet the challenge. We find that cross-sections that include variation in *both* asset risk and asset profitability are able to reproduce most of the observed relationships. Further, these relationships are robust to the inclusion of moderate exogenous variation in debt induced by slow updating of firm capital structures. While we do not claim that the model implies that these findings must hold in general, the parameter heterogeneity we impose does not appear implausible in terms of implied observable firm characteristics.

The results underscore the importance of accounting for both differences in business characteristics and optimal firm policy choice in seeking to understand the cross-section of stock returns. In Section 2 we present a generalized version of the George and Hwang (2009) model. Section 3 analyzes the induced relationships between debt and expected equity returns as firm characteristics vary. Section 4 shows how multidimensional heterogeneity can resolve the empirical puzzles, and assesses the sensitivity of the explanation to the assumption of "purely" endogenous capital structure. A final section summarizes the implications of the work and concludes.

## 2. The model

We consider a somewhat more general version of the model of George and Hwang (2009). While still simplistic, the model is very tractable and allows for a transparent analysis of the capital structure decision and its relation to expected returns.

The model envisions a cross-section of $N$ finite-horizon firms, each having payoffs similar to those in the familiar Merton (1974) model, but in which debt levels are chosen endogenously to trade off distress costs and tax benefits.

The $n$th firm makes a one-time capital structure choice at the time of its birth, $t_0^{(n)}$, to issue an amount $K^{(n)} \geqslant 0$ of zero-coupon debt maturing at a time horizon $t_2^{(n)}$. At that time, the firm will receive a lognormal payoff, $V^{(n)}_{t^{(n)}_2}$, from its operations, and will then liquidate. We normalize the initial value of the payoff process to be $V^{(n)}_{t^{(n)}_0} = 1$ for all $n$ and interpret this as the book value of assets. There are no cash-flows to (or from) investors between birth and liquidation. Nor is the firm permitted to adjust the amount of debt outstanding.

The cross-section of firms is observed at a common time $t_1$ with $t_0^{(n)} \leqslant t_1 \leqslant t_2^{(n)}$ for all $n$. The model thus encompasses the possibility that firms are not all observed at their optimum capital structure, in

[1] Other rational explanations for a negative distress risk premium include that of Garlappi et al. (2008) based on the possibility of cash extraction by equity holders in bankruptcy, and O'Doherty (2010) based on increases in information uncertainty that coincide with distress.

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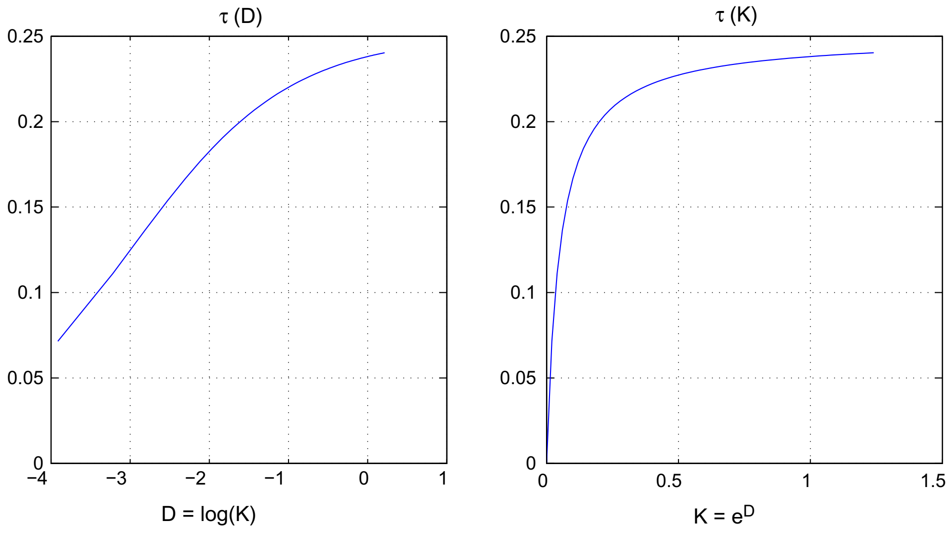


**Fig. 1.** Tax benefit function. The plots show the tax benefit function used in the numerical analysis in Section 3. The face value of debt is $K$ and $D = \log(K)$. The function shown is $A\frac{K^C}{B+K^C}$ where $A = 0.25$, $B = 0.05$, and $C = 1$.

the spirit of Strebulaev (2007). One can view the time from birth to observation as a reduced form parameterization of the severity of a firm's capital adjustment cost: if those costs are low, the firm is likely to be observed close to its optimum value.

We will revert to the dynamics of firm value in a moment. First we describe the initial capital structure decision. For ease of notation we here drop the firm-specific superscript (which apply to all the firm's parameters). Also let $T = t_2^{(n)} - t_0^{(n)}$.

If the payoff at $T$ exceeds the face value of debt, $K$, the firm receives a tax rebate. Here the model imposes a non-standard form to preserve tractability. Specifically, there is assumed to be a function $\tau(D)$, where $D = \log(K)$, which augments the payoff *multiplicatively*, yielding investors $e^{\tau(D)}V_T$. The tax benefit is further contingent because it is assumed to vanish entirely if $V_T < K$. In that case, the firm is bankrupt and experiences additional percentage costs of $c$. The net amount $e^{-c}\,V_T$ is paid to bondholders and equity receives nothing. Notice that, even if $c = 0$, the firm may still only choose a finite amount of debt because, while the tax shield is strictly increasing in $K$, the likelihood of receiving that benefit declines with $K$ since it will be lost in bankruptcy.

We assume that the tax benefit function $\tau(D)$ is non-negative, strictly increasing, and approaches zero as $D \to -\infty$ (i.e. as $K \to 0$). For concreteness, the analysis below assumes the function[2] shown in Fig. 1, which has the properties that (a) it is bounded above; (b) the upper bound is economically large enough to matter (here 25% of firm value); and (c) significant variation occurs over the range of debt values most firms would entertain (e.g. $K = 0.1$–$0.9$). Our conclusions are robust to variations in the precise parametric form. In an appendix available upon request, we verify that the same results hold using unbounded functions as well as additive specifications of the tax benefit.

Contingent on a choice of $K$, the total payoff to investors as a function of the project outcome is shown in Fig. 2.

There are no agency conflicts in the model. The owners of each firm are assumed to maximize the pre-issuance sum of debt and equity values. Each claim is valued under the risk-neutral measure, where the pricing kernel, $m$, evolves in continuous-time according to

$$\frac{dm_t}{m_t} = -r\,dt - \sigma_m dW_t^{(m)}$$

[2] The function plotted is just $A\frac{K^C}{B+K^C}$ where $A = 0.25$, $B = 0.05$, and $C = 1$.

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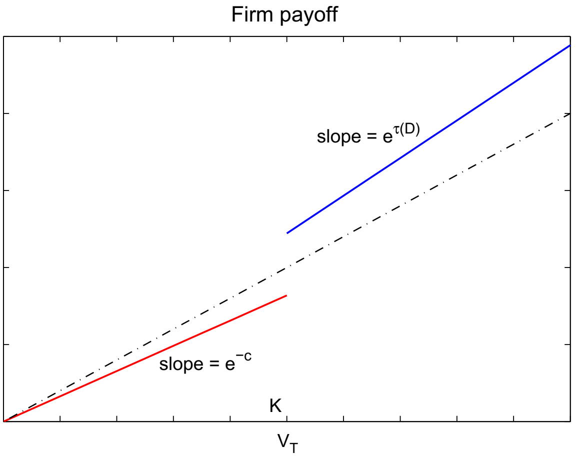


**Fig. 2.** Model payoffs. The solid lines show the payoff to investors (debt plus equity) at time $T$ as a function of the project outcome $V_T$. The firm has debt with face value $K$ and bankruptcy cost parameter $c$.

where the riskless rate $r$ is a constant and $dW^{(m)}$ is a standard Brownian motion. The fundamental value, $V^{(n)}$ of the $n$th firm's investment evolves according to

$$\frac{dV_t^{(n)}}{V_t^{(n)}} = \mu_a^{(n)}\,dt + \sigma_a^{(n)}\,dW_t^{(n)}$$

where the correlation between $dW^{(n)}$ (also a standard Brownian motion) and $dW^{(m)}$ is denoted $\rho^{(n)}$. So the risk-neutral drift of $V^{(n)}$ is $\mu_a^{(n)} - \rho^{(n)}\sigma_a^{(n)}\sigma_m$. We assume throughout that firms have positive systematic risk, i.e., $\rho^{(n)} > 0$ for all $n$.

In Section 3, we specialize the model to case analyzed in George and Hwang (2009) in which the cross-section of firms is observed at the time of their common birth, $t_1 = t_0^{(n)}$. That is, all firms are observed at the moment when they have chosen their optimal $K$. This sub-case reveals the key fundamental relationships at work in the model.

## 3. Equity expected returns and optimal debt

When firms are characterized by different parameters, their optimal debt choice will be co-determined with their equity risk premium, inducing a cross-sectional relation between leverage and expected stock returns. In this section, we explore the effect of varying each of the model inputs in order to see what works and what does not, in terms of reproducing a negative association. The focus will be on two questions: (1) what sign does the parameter variation predict for the relation between optimal debt, $K$, and expected equity returns and (2) how large is the magnitude when the parameter varies over a plausible range? Additional important questions concerning the relationship between default probability and returns, as well as multivariate relationships, are studied in the following section.

Take Proposition 1 in George and Hwang (2009) as a starting point. It establishes that, under the current model, expected excess firm returns rise and optimal debt levels fall as the distress cost parameter, $c$, increases. The logic of the proposition is somewhat subtle. While a high-cost firm will naturally want to decrease the chances of being in bankruptcy (relative to a low-cost one), even having selected a lower debt level, the resulting payoff to investors is still a *more convex* function of the underlying random variable $V_T$, and is thus more exposed to the systematic risk inherent in it. Hence it will still command a higher return premium than a low-cost firm.

Unfortunately the proposition does not address either of the two questions listed above: it says nothing about magnitudes, and it pertains to *firm* – not equity – expected returns. In fact, the conclusions of the proposition are strongly reversed when equity expected returns are considered instead of firm returns.

Fig. 3 shows a typical case (parameters are given in the figure caption). If $\pi_E$ represents the equity risk premium, then $\frac{d\pi_E}{dc} = \frac{\partial \pi_E}{\partial K}\frac{\partial K^*}{\partial c} + \frac{\partial \pi_E}{\partial c}$. The last term is zero because the distress costs are borne entirely by debt holders in default, so there is no direct effect of $c$ on equity payoffs at all. The proposition

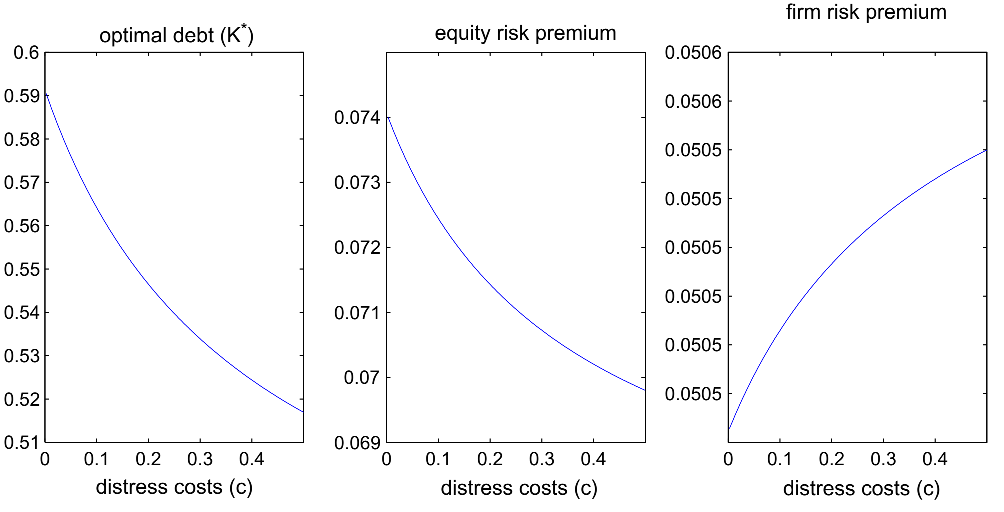


**Fig. 3.** The effect of varying bankruptcy costs. The figure shows optimal debt (left), expected excess returns to equity (center) and expected excess returns to debt plus equity (right) as a function of the parameter $c$. The plots set $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, $\mu_a = 0.12$, $\sigma_a = 0.20$, and $\rho = 0.50$. The tax function parameters are given in the caption to Fig. 1.

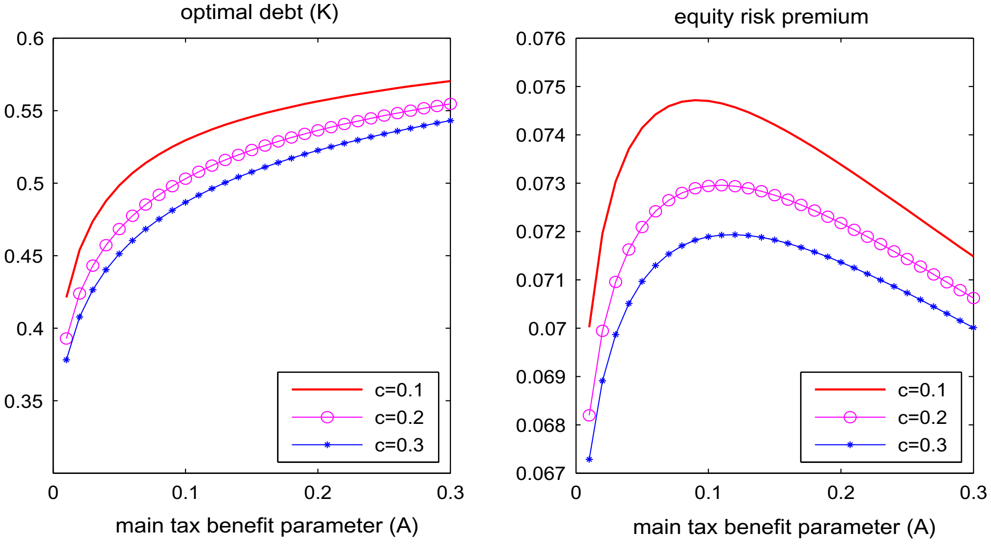


**Fig. 4.** The effect of varying tax benefits. The figure shows optimal debt (left) and expected excess returns to equity (right) as a function of the parameter $A$. The plots set $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, $\mu_a = 0.12$, $\sigma_a = 0.20$, and $\rho = 0.50$. Three values of the bankruptcy cost parameter $c$ are shown. The tax function parameters are given in the caption to Fig. 1.

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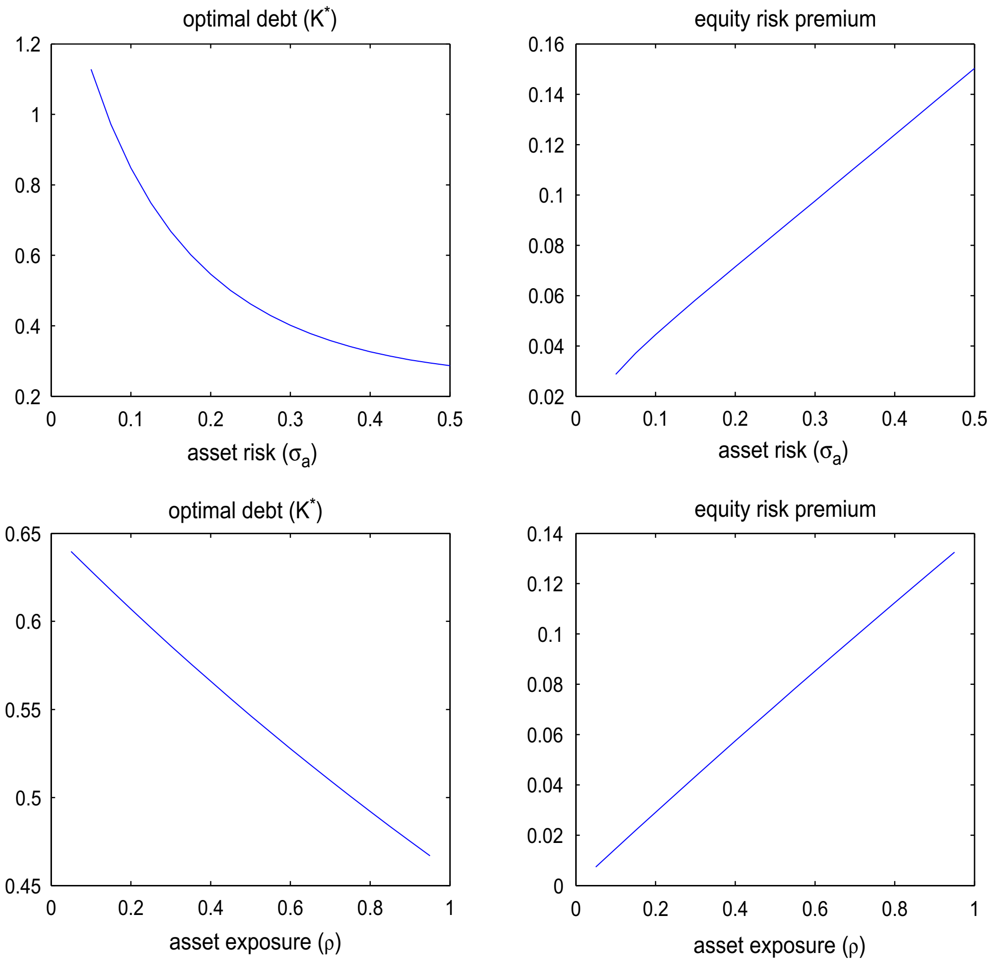


**Fig. 5.** The effect of varying asset risk. The figure shows optimal debt (left), expected excess returns to equity (right) as a function of firm risk parameters. The top panels vary $\sigma_a$ while fixing $\rho = 0.50$. The bottom panels vary $\rho$ while fixing $\sigma_a = 0.20$. All panels use $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, $\mu_a = 0.12$, $c = 0.20$. The tax function parameters are given in the caption to Fig. 1.

shows $\frac{\partial K^*}{\partial c} < 0$, and $\frac{\partial \pi_E}{\partial K} > 0$ is just the standard levering effect of debt. High cost firms which choose lower leverage will thus have safer equity and lower risk premia.[3]

The right panel of Fig. 3 shows a second difficulty with invoking heterogenous distress cost to solve stock expected return puzzles. The magnitude of the induced variation in firm risk premia is too small to be likely to be discernible in empirical tests, even with $c$ running from 0% to 50% of firm value. Intuitively, marginal variations in systematic payoff risk induced through changes in deadweight costs in the future are simply not large enough to have much effect on the firm's overall risk profile.

Now consider the effect of varying the other ingredient in the firm's trade-off calculation: tax benefits. The parameter $A$ controls the magnitude of maximal tax shield, scaling up or down the function plotted in Fig. 1. When varying $A$ as shown in Fig. 4, we see, as expected, that higher tax benefits

[3] Following George and Hwang (2009), the graphs in this section (with one exception) compute the expected equity returns as the log of the equity expected payoff over the price of equity, divided by $T$. That is, these are $T$-horizon expected returns. Using instead *instantaneous* expected returns does not alter any of the conclusions in the analysis.

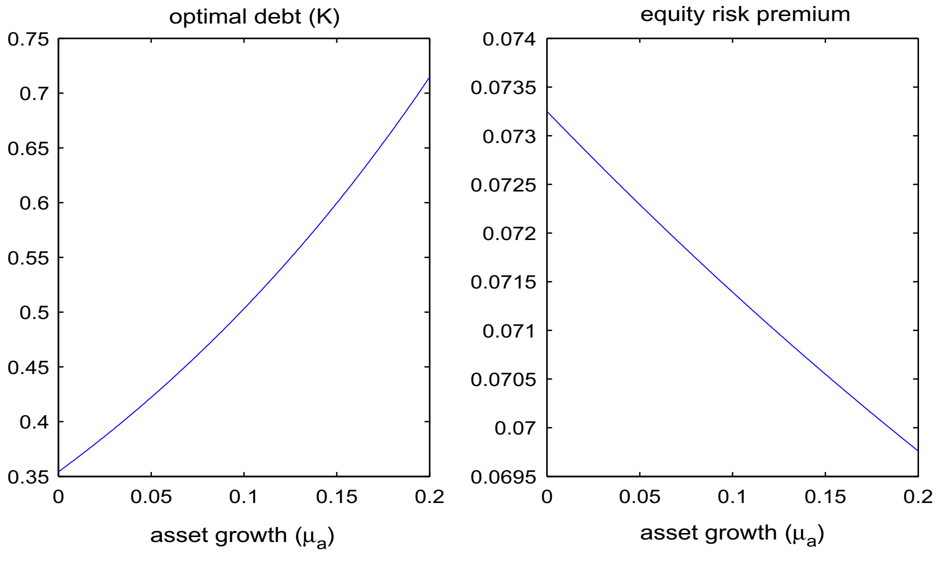


**Fig. 6.** The effect of varying asset growth. The figure shows optimal debt (left), expected excess returns to equity (right) as a function of the parameter $\mu_a$. The plots set $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, $c = 0.25$, $\sigma_a = 0.20$, and $\rho = 0.50$. The tax function parameters are given in the caption to Fig. 1.

always induce higher debt. However, now there is a non-monotonic relation with expected equity returns. When $A$ is greater than approximately 0.10, the equity risk premium has a negative relationship with debt. For lower tax rate firms, the relationship goes the opposite direction.[4]

In either case, the effects are not large: at most about 50 basis points per year over the range of $A$ shown. Moreover, other parameter sets can cause the changeover point (from positive to negative slope of equity returns) to shift. Taken together, then, it seems unlikely (especially considering the positive association induced by $c$) that cross-firm variation in trade-off parameters accounts for the observed negative debt-stock return relation.

Following the intuition that we need riskier firms to choose less debt, the next natural thing to try is simply to vary firms' underlying risk. One can raise risk in the model either by increasing $\sigma_a$ holding $\rho$ fixed (which increases systematic and idiosyncratic risk proportionally) or holding $\sigma_a$ fixed and increasing $\rho$ (which trades off idiosyncratic for systematic risk). Fig. 5 shows that both mechanisms work.

Under this model, high asset-risk firms will choose low debt, but will remain riskier – even in their equity – than low asset-risk firms with more debt. This finding is revealing. Not only might variation in asset risk account for a negative relation between stock returns and leverage, the plots indicate that here the magnitudes can be very large. We will explore this as a potentially realistic explanation for empirical results in Section 4.

Last, we consider the implications of varying the remaining parameters $\mu_a$ and $T$. While not affecting asset risk, these quantities may influence risk premia via their role in determining bankruptcy probabilities.

When varying $\mu_a$, the asset growth rate (or earnings), we do observe an induced negative relation between optimal debt and expected returns. See Fig. 6. Debt rises with $\mu_a$ (since tax benefits are more desirable for more profitable firms), yet expected stock returns fall. This is entirely due to the direct channel: $\frac{\partial \pi_E}{\partial \mu_a} < 0$. Holding $K$ fixed, higher growth de-levers the firm by increasing the value of equity.

[4] Results in Graham (1996) indicate that the distribution of marginal tax rates is bimodal with a substantial mass (around 20%) of observations at zero.

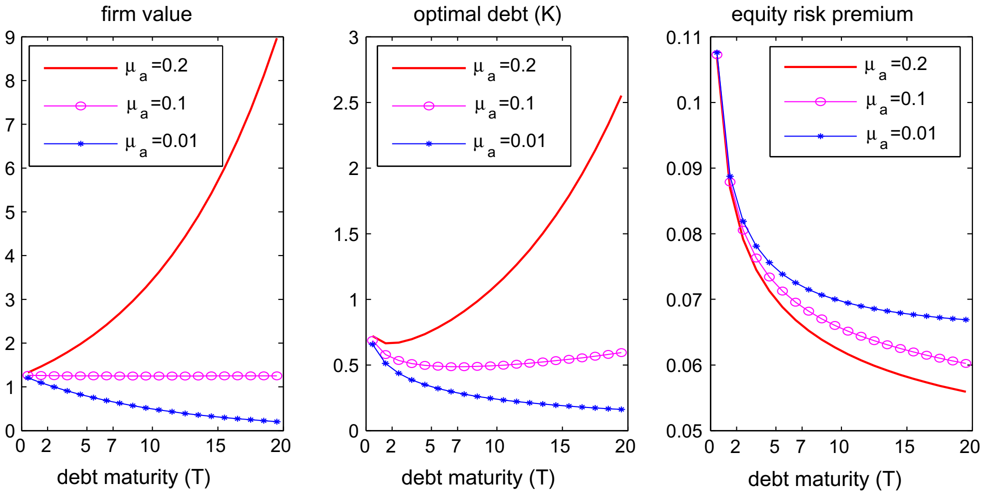


**Fig. 7.** The effect of debt maturity. The figure shows total firm value (debt + equity) (left), optimal debt (center) and expected instantaneous excess returns to equity (right) as a function of the parameter $T$. The plots set $r = 0.05, \sigma_m = 0.50$, $c = 0.25$, $\sigma_a = 0.20$, and $\rho = 0.50$. The tax function parameters are given in the caption to Fig. 1.

While the net effect is not large, it may be empirically important, as firms clearly do differ in their return on assets.

Things are a little more complicated when we consider varying the firm's liquidation date (and debt maturity) $T$. Consider first the case where total firm value is not affected by $T$, which happens when the risk-neutral drift of $V$ is equal to the riskless rate, $r$. An example is shown in the middle line (circles) in Fig. 7. In this case, the optimal level of debt is essentially unchanged with $T$. However, since the risk-neutral drift is still positive, there is again a de-levering effect from equity value increasing with $T$ (as when $\mu_a$ is raised). This produces a decline in equity risk and risk premium with $T$, shown in the right panel. Because $K$ is essentially flat, there is no induced relation between $T$ and the expected returns.[5]

When the risk-neutral growth rate exceeds the riskless rate (see the solid line in the figure), there is the same de-leveraging effect, but also a strong increase in $K$ with $T$ since the firm can increase debt without incurring higher risk of default and because there is more to be gained from tax benefits. The de-levering from the valuation channel is strong enough to dominate the direct levering from increased $K$. This produces a strong negative association between debt and equity expected returns.

Finally, for slowly growing (or less profitable) firms, debt becomes riskier with $T$ (and tax benefits less valuable) and so optimal debt declines. In this case, the valuation channel works to raise equity risk, since longer horizon firms have more (risk-neutral) probability of default. However the decline in $K$ is the stronger effect. So, as with the other cases, equity expected returns fall with $T$. (See the dotted-line plots in the figure.) So now the induced relation with $K$ is positive.

Considering the overall effect of heterogeneity in firm duration, the relationship between $K$ and $\pi_E$ will be ambiguous if (as seems likely) there are both firms with high and low risk-adjusted growth rates – which may either be due to growth rates themselves or their risk adjustment. The analysis suggests that this dimension of heterogeneity may indeed lead to some rich cross-section patterns in firm returns.[6] However, for purposes of accounting for debt-related anomalies, it is not the most straightforward channel.

[5] To ensure that expected excess returns are comparable across firms with differing $T$, the plot shows the instantaneous expected excess returns, instead of the $T$-horizon expected excess returns. See footnote 3. The instantaneous expected return is $\rho\sigma_a\sigma_m V_E'/V_E$, where $V_E = V_E(V_t)$ is the value of equity.

[6] A recent paper by Chen (forthcoming) illustrates that cross-sectional differences in cash-flow duration can account for differences in the magnitude of the value premium across size deciles.

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The analysis here also illustrates that consideration of multidimensional heterogeneity greatly enrich the possible relations that the model can deliver. We now show that some particular parameterizations can indeed account for the relationships reported in the data.

## 4. Resolving the puzzles

Having seen that heterogeneity in some of the model parameters can yield an endogenously negative cross-sectional relationship between debt and expected equity returns, it is important to recognize that this alone does not constitute a sufficient explanation for the empirical puzzles. One reason for this is that a negative association between expected stock returns and *book* debt may not imply a similar association with empirical measures of leverage or distress. Another reason is that empirical studies invariably include other controls which may already capture the heterogeneity.

Specifically, consider the simultaneous relationship between leverage and book-to-market values. High-risk firms will not only have low (book) debt, they will also have low equity values. The equity book-to-market ratio will summarize the firm's risk-adjusted cost of capital, which takes into account the degree of leverage. There thus may be no independent role for debt in explaining equity returns. The puzzles are not about univariate associations. We now demonstrate that multivariate heterogeneity in the model can meet this multivariate challenge.

To break the strong association between book-to-market and expected returns requires additional dimensions of variation in firm parameters that lead to variations more strongly in one characteristic than the other. A natural candidate is firm profitability, which will be reflected in valuation multiples without affecting asset risk exposures. However, it is not obvious whether this will be sufficient, since rising profitability will also lead the firm to optimally choose higher leverage.

Fig. 8 shows how debt, book-to-market, and expected stock returns vary as joint functions of $\mu_a$ and $\sigma_a$.[7] There is clearly a strong negative association between debt and returns. There is also a strong negative relation between debt and the book-to-market ratio ($BE/ME$), but now $BE/ME$ is less clearly linked to expected returns.

Table 1 uses cross-sectional regressions to summarize the model implied joint relationship when a set of diverse firms is observed at the time of their optimization. A simulated cross-section of 4000 independent firms is sampled having $\mu_a$ and $\sigma_a$ drawn from independent uniform distributions over the ranges $[0.00, 0.15]$ and $[0.05, 0.45]$, respectively. The resulting optimal debt levels and $BE/ME$ are then used as independent variables in a regression whose dependent variable is the expected excess return to equity.[8]

With this specification, there is indeed a significant negative role for debt in explaining equity risk premia, even controlling for book-to-market. In fact, the relationship is so strong when book debt, $BD$, itself is an independent variable that the remaining role for $BE/ME$ is (counterfactually) negative. In effect, the regression is using this variable to clean up the nonlinearity in the debt relation. Substituting market leverage, $MD/ME$, for $BD$ restores the correct positive sign on the book-to-market ratio, as shown in the second column. The regression in column 3 follows George and Hwang (2009) and uses dummy variables for firms in the top and bottom quintiles of book debt, which achieves the same result.

A further challenge to the model is to explain the distress-risk puzzle, i.e., to reproduce the negative relationship between expected stock returns and measures of credit risk or default probability. It is important to recognize that this is a distinct goal: in the model, it is not the case that high book debt values coincide with high credit risk. The optimal level of debt to issue is very sensitive to the value that will be realized when it is sold, which drops rapidly with default risk. For most parameterizations, this leads to an *inverse* relation between debt and credit risk.

[7] We define "book equity" as cost of assets – which is 1 for all firms – minus face value of debt. Since the debt has zero coupon, one could argue that the "book value" of debt should be its present value, with book equity adjusted accordingly. The results below are not sensitive to this alternative definition.

[8] Note that this regression does not mimic empirical work in that we use the true expected return on the left-hand side, not observed (noisy) returns. The $t$ statistics in the table are thus not comparable to those using real data. They can be viewed as a measure of the relative contribution of each variable in the regressions.

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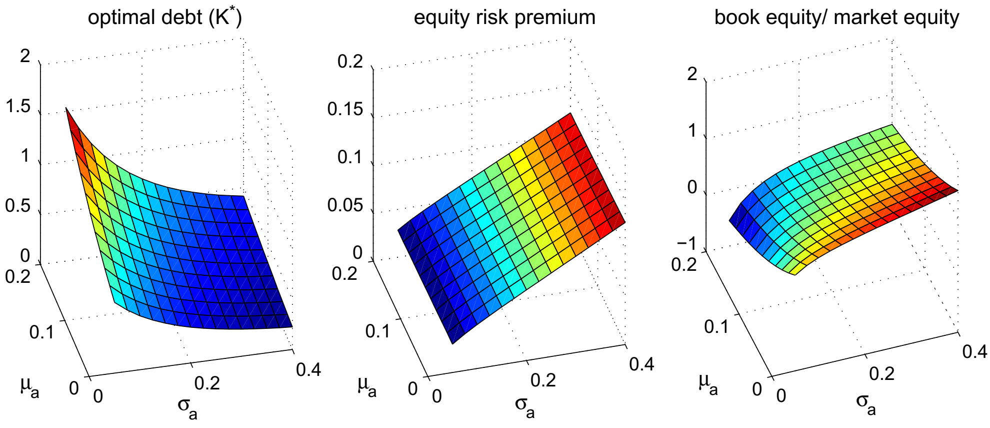


**Fig. 8.** Simultaneous variation in risk and profitability. The figure shows optimal debt (left), expected excess returns to equity (center) and the ratio of book-to-market equity (right) as a function of $\mu_a$ and $\sigma_a$. All panels use $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, $c = 0.20$, and $\rho = 0.50$. The tax function parameters are given in the caption to Fig. 1.

**Table 1**
Expected return regressions with two-dimensional variation.

| Variable | 1 | 2 | 3 |
|---|---|---|---|
| Book-to-market | −0.0305<br>(21.7) | 0.0229<br>(30.2) | 0.0163<br>(11.2) |
| Book debt | −0.1843<br>(74.1) | | |
| Market leverage | | −0.0852<br>(82.2) | |
| High debt | | | −0.0352<br>(34.3) |
| Low debt | | | 0.0279<br>(28.2) |

The table shows the results of OLS regressions of expected excess returns to equity on firm-specific characteristics based on a sample of 4000 firms in which $\mu_a$ and $\sigma_a$ are drawn from independent uniform distributions over the ranges [0.00, 0.15] and [0.05, 0.45], respectively. The numbers in parentheses are OLS $t$ statistics. Firms are assumed to be observed at the time of optimization. The variables "high debt" and "low debt" are dummies for whether book debt is in the top or bottom quintile of firms in the sample. The parameters common to all firms are $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, $c = 0.20$, and $\rho = 0.5$. The tax function parameters are given in the caption to Fig. 1.

Addressing the credit-risk and debt level findings simultaneously requires introducing another degree of variation in the cross-section of firms. Table 2 shows regression results from a simulated sample of firms whose profitability, systematic risk, and idiosyncratic risk all vary. (Specifically, the sample draws $\mu_a$, $\sigma_a$, and $\rho$ from independent uniform distributions over ranges [0.0, 0.20], [0.05, 0.35], and [0.10, 0.90], respectively.) As in the earlier sample, the regression reproduces a positive relationship with *BE*/*ME* and negative one with *BD*. In addition, credit risk, as measured by *EDF* (the probability of the firm defaulting) has a significantly negative relationship with expected stock returns. The latter is not subsumed by book leverage,[9] but constitutes a third dimension of variation in the cross-section.

[9] George and Hwang (2009) find that some proxies for default risk are rendered insignificant when controlling for book debt.

The exercise here shows that the model can accommodate multiple features of the data. The variation in firm characteristics of the simulated cross-section is also not unreasonable. Histograms in Fig. 9 display the induced levels of debt, book-to-market ratios, and equity risk premia implied by the distribution from which the parameters were drawn. Evidently the findings are not driven by implausible heterogeneity assumptions.

However we cannot claim that the relationships we have re-created are necessary implications of the model. Indeed, because of the high correlation among the induced firm characteristics, one can

**Table 2**
Expected return regressions with three-dimensional variation.

| Variable | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Book-to-market | 0.0357 | 0.0501 | 0.0407 | 0.0655 | 0.0357 |
| | (12.7) | (36.1) | (18.0) | (46.7) | (16.2) |
| Book leverage | −0.0508 | | | | |
| | (10.7) | | | | |
| Market leverage | | −0.0403 | | | |
| | | (18.8) | | | |
| High debt | | | −0.0084 | | −0.0157 |
| | | | (4.8) | | (8.2) |
| Low debt | | | 0.0253 | | 0.0341 |
| | | | (14.3) | | (18.9) |
| High DP | | | | −0.0142 | −0.0216 |
| | | | | (10.7) | (16.3) |
| Low DP | | | | −0.0009 | 0.0013 |
| | | | | (0.7) | (0.9) |

The table shows the results of OLS regressions of expected excess returns to equity on firm-specific characteristics based on a sample of 4000 firms in which $\mu_a$,$\sigma_a$, and $\rho$ are drawn from independent uniform distributions over the ranges [0.0, 0.20], [0.05, 0.35], and [0.10, 0.90], respectively. The numbers in parentheses are OLS $t$ statistics. Firms are assumed to be observed at the time of optimization. The variables "high debt" and "low debt" are dummies for whether book debt is in the top or bottom quintile of firms in the sample. The variables "high DP" and "low DP" are dummies for whether the probability of default is in the top or bottom quintile of firms in the sample. The parameters common to all firms are $r = 0.05$, $\sigma_m = 0.50$, $T = 4$, and $c = 0.20$. The tax function parameters are given in the caption to Fig. 1.

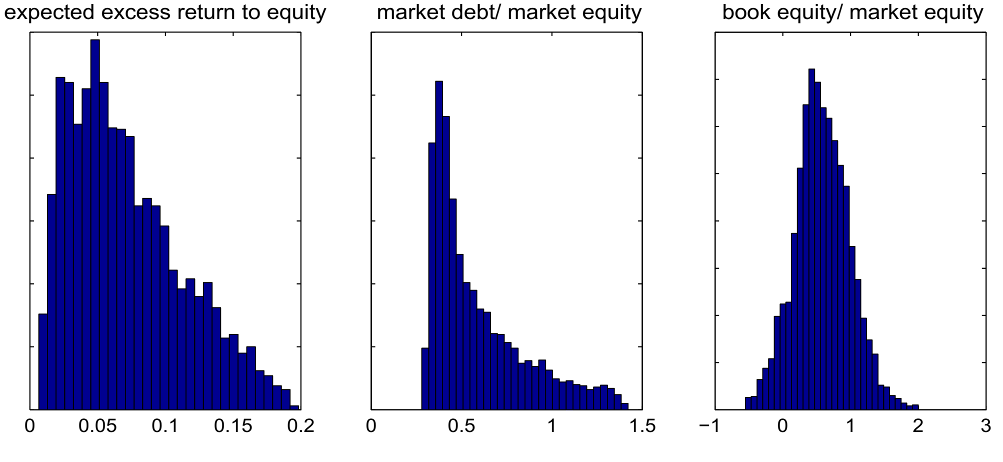


**Fig. 9.** Firm characteristics.The figure shows histograms of firm characteristics for the simulated sample used in Table 2.

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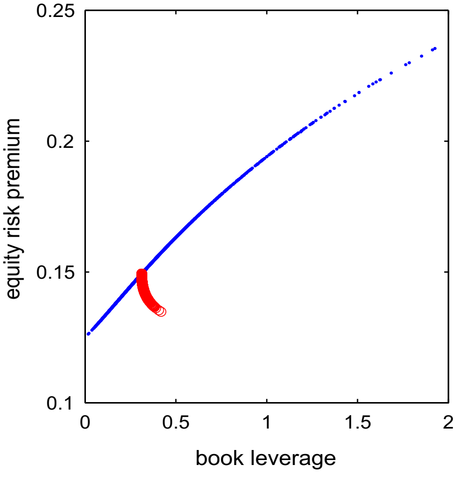


**Fig. 10.** Exogenous vs endogenous leverage variation. The dotted-lines show the equity risk premium versus book leverage and market leverage for a sample of 4000 identical firms that are born at different times, $t_0^{(n)}$, and observed at a common time, $t_1$, when all have $T = 4$ years to maturity. The time length $t_1 - t_0^{(n)}$ is drawn from an exponential distribution with mean 2.0. The circles to the lower left of each panel show the relationship when all firms are observed at birth. All firms have $\mu_a = 0.15$, $\sigma_a = 0.2$, $\rho = 0.5$, $c = 0.2$.

cause many of the regression coefficients to switch signs using sample designs that also seem defensible. Based on numerical exploration of the three-dimensional variation used in Table 2, getting all the correct relationships to be significant in specification 5, for example, appears to require that the sample contains both low growth firms ($\mu_a < 0.05$) and low correlation firms ($\rho < 0.25$).

One dimension of generality that we can readily check is the sensitivity of the results to the assumption that firms are observed precisely at the time of optimization. This assumption effectively maximizes the endogeneity of debt, which is clearly the driving force behind our findings. However, as emphasized in the dynamic capital structure literature (e.g., Strebulaev (2007)), firms are in reality slow to re-optimize due to financing frictions. Thus some exogenous variation in debt seeps into the cross-section.

Fig. 10 shows how this hurts our argument. We imagine a cross-section of otherwise identical firms, that each optimized at some time $t_0^{(n)}$ in the past, but which are now observed at a common time $t_1$ at which their debt has 4 years remaining to maturity. We draw $t_1 - t_0^{(n)}$ from an exponential distribution with mean 2.0 (years) and then draw the random subsequent changes in fundamental value $V_t^{(n)}$ over that interval. (All firms have $V = 1$ at birth, as before.) The plots show the relation between expected equity returns and leverage (book and market values) both at the time of initial optimization (circles) and at the time of observation (dots). In the cross-section of firms at optimization there is some variation in expected returns, due to firms' differing durations, but all are between 14% and 15%. This variation is positively associated with market leverage, but negatively associated with book leverage due to the endogeneity mechanism.[10]

At $T_1$ however, when some time has passed, there is a wide variation in expected returns which (since the firms are otherwise equal and their parameters have not changed) can only be due to differences in $V_t/K$: firms that experienced positive news have effectively de-levered, whereas effective leverage has increased strongly for bad-news firms. As a result of this exogenous variation (i.e., unrelated to the firm's initial choice) there is now a strong positive association between equity risk premium and market *and* book leverage.

[10] For this exercise, we interpret the ''book'' value of assets at the observation time to be the fundamental value $V_t$. Book leverage at $t$ is defined as $K/V_t$, and book equity is $V_t - K$.

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**Table 3**
Cross-sections of off-optimum firms.

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *Panel A* | | | | | |
| Book-to-market | 0.0556 | 0.0569 | 0.0367 | 0.0614 | 0.0370 |
| | (21.2) | (37.7) | (15.5) | (44.0) | (15.3) |
| Book leverage | −0.0152 | | | | |
| | (3.6) | | | | |
| Market leverage | | −0.0214 | | | |
| | | (9.8) | | | |
| High debt | | | −0.0127 | | −0.0109 |
| | | | (7.1) | | (5.6) |
| Low debt | | | 0.0277 | | 0.0275 |
| | | | (15.1) | | (14.3) |
| High DP | | | | 0.0037 | −0.0016 |
| | | | | (2.7) | (1.2) |
| Low DP | | | | −0.0070 | −0.0049 |
| | | | | (5.0) | (3.4) |
| *Panel B* | | | | | |
| Book-to-market | 0.0360 | 0.0502 | 0.0143 | 0.0453 | 0.0172 |
| | (16.1) | (32.3) | (8.0) | (39.5) | (9.4) |
| Book leverage | −0.0251 | | | | |
| | (6.8) | | | | |
| Market leverage | | 0.0056 | | | |
| | | (2.7) | | | |
| High debt | | | −0.0220 | | −0.0171 |
| | | | (14.0) | | (10.1) |
| Low debt | | | 0.0397 | | 0.0368 |
| | | | (24.4) | | (21.8) |
| High DP | | | | 0.0117 | 0.0030 |
| | | | | (8.3) | (2.1) |
| Low DP | | | | −0.0158 | −0.0097 |
| | | | | (11.1) | (6.9) |

The table shows the results of OLS regressions of expected excess returns to equity on firm-specific characteristics based on a samples of 4000 firms whose characteristics are given in the caption to Table 2. In the Panel A, the time since optimization is exponentially distributed with mean 1.0. In the Panel B, the time since optimization is exponentially distributed with mean 2.0.

The cross-sections we generated above for Table 2 had substantially stronger endogenous leverage effects than the one in this figure. But clearly if enough time has gone by since optimization, they too will be characterized mostly by exogenous leverage variation, and the negative association with expected returns will be lost. The question is how much time is too much.

Table 3 repeats the regression tests in Table 2 when we let the time since optimization be exponentially distributed with means 1.0 (Panel A) and 2.0 (Panel B). For purposes of comparison, we leave the sample size unchanged at 4000.

The table reveals that the negative relation between book leverage and expected returns (controlling for book-to-market) is remarkably robust, and survives even in the second panel where the expected lag since optimization is 2 years. The negative relation with market leverage survives with one year of expected observation lag, but not with two. Both cross-sections lose the desired relationship with the default probability indicators. Still, it is fair to conclude that the assertion that the major features of the debt puzzles may be explained by cross-sectional heterogeneity does not depend strongly on the absence of any exogenous leverage variation.

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## 5. Conclusion

We concur with George and Hwang (2009) that endogenous leverage choice and rational asset pricing may imply a negative and significant relation between debt (or leverage or distress risk) and expected equity returns. The model they propose offers a straightforward way to quantify these relationships, which can be subtle, especially in the presence of related variables.

Using that model, we illustrate that heterogeneity in bankruptcy costs is not a sufficient condition. (In fact, the relationship with the equity risk premium goes the wrong way.) Variation in tax benefits and in firm duration can, in certain circumstances deliver a negative association, but may also imply the opposite.

We show that simultaneous variation in firm profitability and risk can not only produce the desired debt effects, but it can do so while controlling for differences in book-to-market ratios. When both systematic and idiosyncratic risk vary, the model can also lead to a negative relation between default probability and expected stock returns. The negative relations between equity expected returns and book or market leverage are preserved when allowing for exogenous perturbation of leverage induced by financing frictions.

To date, most rational asset pricing models that seek to explain cross-sectional patterns in observed returns have not invoked firm heterogeneity. The alternative strategy has been to model firms that are *ex ante* identical, but that experience differing histories of productivity shocks or investment opportunities. This approach has much to recommend it, and has yielded many insights, especially with regard to the dynamic evolution of firms' risk profiles. (Obreja (2006) and Gomes and Schmid (2010) analyze the leverage-return relationship in this general framework.) However, based on the analysis here, it is clear that also taking into account simple, static variation in cash-flow characteristics across different business types has great potential to add to our understanding of observed relationships between asset returns and other company characteristics. An important challenge, moving forward, is to establish the rules of the game for this type of exercise by parameterizing the actual joint distribution of asset characteristics.

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## Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.frl.2010.12.003.

## References

Chen, Huafeng, forthcoming. Firm life expectancy and the heterogeneity of the book-to-market effect. Journal of Financial Economics.
Garlappi, Lorenzo, Shu, Tao, Yan, Hong, 2008. Default risk, shareholder advantage and stock returns. Review of Financial Studies 81, 2743–2778.
George, Thomas J., Hwang, Chuan-Yang, 2009. A resolution of the distress risk and leverage puzzles in the cross section of stock returns. Journal of Financial Economics 96, 56–79.
Gomes, Joao F., Schmid, Lukas, 2010. Levered returns. Journal of Finance 65, 467–494.
Graham, John R., 1996. Debt and the marginal tax rate. Journal of Financial Economics 41, 41–73.
Merton, Robert C., 1974. On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance 29, 449–470.
Obreja, Ilujan, 2006. Financial Leverage and the Cross-section of Stock Returns. Carnegie Mellon University Working Paper.
O'Doherty, Michael S., 2010. Information Risk, Conditional Betas, and the Financial Distress Anomaly. University of Iowa Working Paper.
Strebulaev, Ilya A., 2007. Do tests of capital structure theory mean what they say? Journal of Finance 62, 1747–1787.

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