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Volume, liquidity, and liquidity risk ☆

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Abstract

Many classes of microstructure models, as well as intuition, suggest that it should be easier to trade when markets are more active. In the data, however, volume and liquidity seem unrelated over time. This paper offers an explanation for this fact based on a simple frictionless model in which liquidity reflects the average risk-bearing capacity of the economy and volume reflects the changing contribution of individuals to that average. Volume and liquidity are unrelated in the model, but volume is positively related to the variance of liquidity, or liquidity *risk*. Empirical evidence from the U.S. government bond and stock markets supports this new prediction.
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1. Introduction

Recent empirical studies of liquidity dynamics have turned up a surprising negative result: higher volume does not necessarily lead to more liquid markets. In annual vector autoregressions using the Dow Jones 30 industrial stocks, Jones (2002) finds no significant effect of changes in turnover on changes in bid-ask spreads. Using monthly aggregate stock market data, Fujimoto (2004) finds mostly insignificant impulse responses to turnover shocks for several liquidity measures. Evans and Lyons (2002) and Galati (2000) find no association between liquidity and level of activity on the foreign exchange market, while Danielsson and Payne (2002) find a *negative* relationship. Similar negative findings are reported in Foster and Viswanathan (1993) and Lee, Mucklow, and Ready (1993) for individual stocks.[1] In the U.S. Treasury market, Fleming (2003) finds that neither trading volume nor trading frequency are consistently correlated with price impact or bid-ask spreads.

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[1] A positive association between trading activity and volume is sometimes observed. There are common time-of-day and day-of-week effects in spreads or depth and volume. See Chordia, Roll, and Subrahmanyam and Barclay and Hendershott (2004), for example. Lipson and Mortal (2007) find that bidder firms experience increases in volume and liquidity following takeovers.

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These results seem to challenge the basic intuition that it ought to be easier to trade in more active markets. This intuition certainly seems to find support cross-sectionally. Larger, more active markets are usually more liquid. Indeed, this was the original finding of Demsetz (1968), which is sometimes viewed as the starting point of the field of market microstructure. More frequently traded stocks have lower bid-ask spreads.

To Demsetz, the result was an unsurprising consequence of competitive intermediation: higher transaction demand leads to more profit for dealers and hence cheaper provision of liquidity services. Further, lower costs should naturally elicit more trade, as in any other product market. Underlying this view, of course, is the conception of liquidity as the output of a sector with access to a particular intermediation technology.

However, the same conclusion applies for different reasons in some classical models of asymmetric information. In Kyle (1985), equilibrium in the game between informed traders and liquidity suppliers requires that informed demand (and hence volume) scales with uninformed demand, while illiquidity (Kyle's lambda) is inversely proportional to the scale of uninformed demand because more noise makes total order flow less informative. Hence more volume means higher liquidity. This is a comparative static result. However, the logic is borne out in dynamic extensions such as Admati and Pfleiderer (1988) and Foster and Viswanathan (1990). Again, when uninformed traders are allowed to respond to variations in liquidity, the relationship is strengthened.[2]

Finally, a third trading paradigm, search models, also implies that more active markets are more liquid. When there is more search activity for whatever reason, then liquidity—measured by the opportunity cost of the searching time—will be shorter, almost by definition (see Lippman and McCall, 1986).

The lack of a *dynamic* relationship between liquidity and volume thus seems to pose something of a challenge, both to intuition and to several classes of model. The consistency of the non-finding across several types of asset, market structure, and frequency suggests that there is indeed something to explain here. Moreover, all the empirical studies cited above control for things such as returns and volatility. So it is not the case, for example, that liquidity fails to rise with volume because volume rises with uncertainty.

Understanding liquidity dynamics is important for a number of reasons. Since liquidity directly determines the feasibility and costs of dynamic trading strategies, any investor or institution that needs to implement such a strategy must quantify the liquidity risk involved. Because investors care about it, studying the consequences of liquidity risk has become a central topic in asset pricing (see Pástor and Stambaugh, 2003; Acharya and Pedersen, 2004). Finally, liquidity risk is important from a policy perspective because of the danger posed by large drops in liquidity, which may lead to price distortions, disruptions in risk transfer, and possibly inefficient liquidation of real investments.

Perhaps especially for this latter reason, a crucial aspect of our understanding of liquidity dynamics is pinning down the role played by intermediaries or "liquidity providers." It is here that the empirical volume–liquidity results are directly relevant. By falsifying a basic intuition, they seem to call into question the view of liquidity as a service output provided by a segmented sector. This view underlies much policy analysis of market fragility, which regards the capital constraints of intermediaries as the key determinant of liquidity risk.

An alternative view, in which intermediaries play no role, models liquidity as the average willingness of the market as a whole (or a representative agent) to accommodate trade at prevailing prices (Pagano, 1989). This willingness may fluctuate as the underlying state of the economy changes. In general, agents are more flexible and asset prices respond less to trade demand when a marginal perturbation to their portfolios has a low impact on their intertemporal marginal rate of substitution (discount rates). Johnson (2006) develops the calculation of this liquidity in representative agent economies and illustrates its endogenous dynamics in several examples. By definition, however, there is no trade in representative agent economies, meaning that the examples in that paper can shed no light on the connection between volume and liquidity.

This paper extends Johnson (2006) by computing the equilibrium price-response measure of liquidity in a simple multi-agent economy with trade. The derivation explicitly connects the (shadow) illiquidity characterizing a representative agent's demand curve for shares with the actual trade impact costs that

[2]Not all asymmetric information models support this intuition. In Spiegel and Subrahmanyam (1992), uninformed volume raises volatility, which hurts liquidity. In Easley and O'Hara (1992), informed traders have no timing or quantity choice, so higher volume necessarily means more information risk, which lowers liquidity.

affect the portfolio decisions of disaggregated agents. Hence, the paper presents further evidence on the usefulness of the underlying concept of price elasticity by using it to show exactly how liquidity and volume are jointly determined.

The setting is a frictionless model of stochastically participating agents whose arrivals and departures generate (exogenous) trade demand. Liquidity fluctuates as the aggregate risk-bearing capacity of the economy changes due to changes in total participation. Despite its simplicity, the model delivers an aggregate liquidity process that behaves realistically, covarying positively with asset prices and negatively with volatility, for example. Other aggregate relationships are also sensible: volume covaries positively with volatility, and volatility covaries negatively with returns (see Appendix B).

The model implies, however, that there should be no relationship between the levels of liquidity and volume. Volume is simply driven by the degree of flux of the population, which is independent of the level of participation. The usual intuition that it is easier to transact in a busy market fails because high turnover does not make providing liquidity any less costly to the remaining agents. Trade *per se* has no real effects.

While accounting for the non-relationship between volume and liquidity resolves a conceptual puzzle, the model goes further and delivers a novel positive implication. If volume reflects compositional rearrangement, then large increases *or decreases* in willingness to bear risk should *both* necessitate high volume. Conversely, higher trading intensity makes higher population flux more likely—in either direction. Hence volume should be associated with the *second* moment of liquidity changes, that is, with liquidity risk. Moreover, this relationship should hold independent of any changes in volatility.

While alternative explanations for the lack of a (first-moment) relationship between liquidity and volume can be found, it is not at all clear that any such alternative would also make the second-moment prediction I suggest here. If liquidity provision were non-competitive, for example, higher volume might just lead to higher intermediary profits without any reduction in trading costs. In that case, however, there would also be no connection between volume and liquidity risk. If asymmetric information risk rises with volume but the signal-to-noise ratio in order flow is constant (as happens in Bernhardt and Hughson, 2002, for example), then liquidity is unaltered in either direction when volume increases. In short, the volume-liquidity risk hypothesis is not necessarily unique to this paper, but it does not seem to have been previously articulated.

I examine this hypothesis in the context of the U.S. government bond and stock markets, and the evidence is supportive. The effect appears to be both significant and robust. An additional contribution of the work, then, is to sharpen our understanding of the drivers of liquidity fluctuation.

The model ignores information asymmetries and many other important factors that affect trading in real markets. Moreover, the focus is exclusively on dynamic patterns, leaving many interesting cross-sectional questions unaddressed. Nevertheless, the findings may have important implications. They suggest that one widely accepted determinant of liquidity risk—the organization and financial position of intermediaries—may not matter as much as one might think. At the same time, the argument suggests that another factor usually thought to indicate market health—higher turnover—may actually be associated with increased liquidity risk.

The rest of the paper is organized as follows. Section 2 introduces the model and establishes its predictions. Section 3 presents numerical evidence on the model's main properties. Section 4 presents the empirical evidence. A final section summarizes the paper's contribution and implications.

2. A model of volume and liquidity

The plan of this section is to describe a simple economy in which agents trade assets, quantify trading demands, solve the model, characterize liquidity, and deduce the implied joint behavior of liquidity and volume. While the setup is necessarily simplified, I will argue that the conclusions are likely to be robust in some important respects.

To start, consider a continuous-time economy populated by N_t agents with constant absolute risk aversion (CARA) preferences. There is one consumption good, one risky asset, and a riskless technology for storing the good with constant return r. I will denote the total stock of stored goods as $X_t^{(0)}$ and sometimes refer to this simply as (aggregate) cash holdings. The supply of the risky asset is fixed, and the number of shares outstanding will be denoted $X^{(1)}$. Each share receives a continuous stream, Θ_t, of the consumption good per unit time. The dividend rate evolves as an arithmetic Brownian motion, $\mathrm{d}\Theta_t = \mu_\Theta\,\mathrm{d}t + \sigma_\Theta\,\mathrm{d}Z_t$.

All agents have the same preferences over consumption flows, c_t, characterized by the instantaneous utility $-e^{-\gamma c_t}$ and rate of time preference ρ. Agents have exponentially distributed lifespans, dying at rate μ_1 per unit time. New agents are born at rate $\lambda(N)$, which is a function (to be described) of the total population size. The birth and each of the death processes are mutually independent and independent of the dividend innovations. Formally, the birth and death increments are independent Poisson processes, A_t and D_t, respectively, having state-dependent intensities $\lambda(N)$ and $\mu(N) \equiv \mu_1 N$, so that

$$\mathrm{d}N = \mathrm{d}A - \mathrm{d}D$$

gives the population increment. Note that the process is nonnegative because only instantaneous transitions of ± 1 can occur in continuous time, and $\mu(0) = 0$.[3] Rather than birth and death, one could also simply interpret the component processes as describing the arrival and departure of agents from the asset market for unmodelled reasons.

2.1. Trading arrangements and volume

The model says nothing about why agents enter and exit the market. It therefore cannot shed any light on when and why people trade. What it does provide is a straightforward framework for quantifying how much they trade and how turnover and liquidity are jointly determined.

When agents are born they have no shares of the risky asset and choose their holdings optimally. Upon death, agents' shareholdings are liquidated. It will not be necessary to specify what happens to the proceeds (nor is it necessary to specify the initial riskless endowment of the newly born) because cash holdings do not enter into the determination of any equilibrium quantities.[4] Shares are traded in frictionless, competitive markets; there are no transactions costs of any kind.

Since at each point in time every agent faces the same decision problem, all will follow identical consumption and portfolio policies. In particular, in equilibrium each will have to hold $X^{(1)}/N_t$ shares of the risky asset. Hence, this is also the amount that will be purchased on every arrival and the amount sold on each departure. Moreover, in continuous time, each arrival and departure event happens at a distinct point in time, so there is no overlap of trade demand. This establishes a very simple expression for volume.

Proposition 2.1. *Let A_t and D_t denote the cumulative arrival and departure processes. Let $\mathscr{V}(t_1, t_2)$ denote the number of shares traded in the interval $[t_1, t_2]$ and let $\mathscr{T} = \mathscr{V}/X^{(1)}$ be the corresponding turnover.*

Then, if trade takes place continuously,

$$\mathscr{T}(t_1, t_2) = \int_{t_1}^{t_2} \frac{\mathrm{d}A_t}{N_t} + \int_{t_1}^{t_2} \frac{\mathrm{d}D_t}{N_t}.$$

Here the increments in the numerator, $\mathrm{d}A$ and $\mathrm{d}D$, will scale with the respective arrival and departure intensities. If, as is assumed below, these intensities scale linearly in the population size,[5] then the expression indicates that, while turnover will scale with the intensity-per-agent, μ_1, it will not vary with N, which cancels out. A higher population means more trades, but each of fewer shares. In this model, then, turnover is approximately *i.i.d.*

Since the simple characterization in the proposition relies on what is arguably an unrealistic feature of continuous time, an alternative formulation is to view markets as clearing at discrete time points $\{t_n\}$ (while the

[3] The population N is an example of a continuous-time birth-and-death process that can also be defined directly via the infinitesimal generator

$$\begin{bmatrix} -\lambda(0) & \lambda(0) & 0 & \cdots & & \\ \mu(1) & -(\lambda(1)+\mu(1)) & \lambda(1) & 0 & \cdots & \\ 0 & \mu(2) & -(\lambda(2)+\mu(2)) & \lambda(2) & 0 & \cdots \\ \vdots & & \ddots & & & \end{bmatrix}.$$

See Karlin and Taylor (1975, Chapter 4), for example.

[4] This is, of course, a specialized property of CARA utility.

[5] The arrival intensity $\lambda(N)$ will deviate slightly from linearity at low values of N.

underlying processes still evolve in continuous time) such that trade demands are offset to the extent possible. This is equivalent to stipulating that the balance of the population, which has no need to trade, trades no more than is necessary. (Under the previous assumptions, every trade requires an adjustment of every agent's portfolio.) Interestingly, this convention also leads to a simple formula.

Proposition 2.2. *Suppose trade takes place at times* $\{t_n\}$, *let* $A_n \equiv \int_{t_{n-1}}^{t_n} \mathrm{d}A_t$ *denote the arrivals in the interval* $[t_{n-1}, t_n]$, *and define* D_n *likewise.*

Then, if buyers and sellers are matched to the maximal extent possible,

$$\mathscr{T}(t_n) = \max\left\{\frac{A_n}{N_{t_n}}, \frac{D_n}{N_{t_{n-1}}}\right\}.$$

Note. *All proofs appear in Appendix A.*

This expression for turnover can be understood as just saying that the total amount of required trade (roughly $\frac{A_n}{N_{t_n}} + \frac{D_n}{N_{t_{n-1}}}$) can be reduced by the minimum of the two demands, which can be crossed between entering and exiting agents. Whether the continuous-trade expression is better than the discrete-trade one is an empirical question. Both will be explored below.

Note that, under either arrangement, volume does not reflect the total change ΔN_t in the population. One could have large volume (over a given period) and end up with the exact same population level, and hence the same economy, as if no volume had occurred. Likewise, given a large volume observation, one cannot infer anything about the *direction* of likely change in N_t.

Instead of information about the change, volume captures something closer to the physical concept of flux: the total (gross) flow into and out of the population. This, in turn, captures the degree to which the individuals within the population have been rearranged. While this notion may seem specific to the way that the present model induces trade, it is more general than that. The classic model of Campbell, Grossman, and Wang (1993) generates trade between CARA agents via idiosyncratic shocks to individual risk aversion coefficients (i.e., $\gamma_t^{(i)}$). There, too, high volume occurs *not* when there are large changes to the overall state of the economy (determined by the average $\gamma^{(i)}$), but when the mean change in individuals—relative to the average—is high. A similar result can be derived in the model of Lo, Mamaysky, and Wang (2004), where trade occurs because of idiosyncratic shocks to private risk exposures. Indeed, in the limit of an infinite population, *all* moments of the cross-agent distribution of this exposure are constant through time, by a law of large numbers. Yet volume still occurs because of the repositioning of individual agents within this distribution. To a large extent, the conclusions below are driven by the intuition common to these examples.

2.2. Prices and liquidity

The next result describes asset demands and equilibrium prices in the economy. I treat the continuous trade case for analytical simplicity.[6] To further simplify, I choose the arrival intensity function, $\lambda(N)$, in a manner that makes agents indifferent to population jump risk. As a practical matter, this is essentially the same as taking the inverse population size $1/N_t$ to be a martingale.

Proposition 2.3. *Assume trading takes place continuously, and let* $\mu(N) = N\mu_1$ *be the aggregate departure intensity. There exists an arrival intensity function* $\lambda(N)$, *characterized in the appendix, for which the following asset demands* $\alpha(P)$ *and prices* P *constitute an equilibrium of the economy*:

$$\alpha(P) = \frac{\Theta + \mu_\Theta/r - rP}{\gamma\sigma_\Theta^2/r}, \tag{1}$$

[6] The existence of analogous price and demand expressions when trading takes place discretely is still conjectural. However, there is a degree of freedom in the argument—the construction of the arrival intensity function—which is chosen to lead to the forms in the proposition. A similar choice can be made with discrete trade. Only the machinery to formally characterize the solution is absent. I have also verified that the constructed continuous-time solution for $\lambda(N)$ together with the resulting price expression do indeed solve numerically the discrete-trade Bellman equation to a high degree of accuracy.

$$P(\Theta, N) = \frac{\Theta}{r} + \frac{\mu_\Theta}{r^2} - \frac{\gamma\sigma_\Theta^2}{r^2 N} X^{(1)}. \tag{2}$$

Moreover, the function $\lambda(N)$ *satisfies*

$$\frac{\lambda(N)}{(N+1)\mu_1} \sim 1 + O(N^{-3}).$$

These expressions for demand and prices are exactly those that would obtain with a nonstochastic CARA population. That is by design because the goal here is to study volume and liquidity in a familiar setting and not be distracted by new effects coming from a particular choice of population process. It turns out that the population dynamics required to achieve this (determined by $\lambda(N)$ and $\mu(N)$) are not unnatural. The departure rate of each agent is constant. And the arrival rate per person is almost the same. From the last statement in the proposition, $\lambda(N)/(N+1) \approx \mu(N)/N = \mu_1$. The slightly higher arrival rate means that the population will, on average, grow slightly. This is a consequence of the technical need that $1/N$—a convex function of N—be unpredictable. But it is not unrealistic.

A cost of unpredictable changes in N, however, is that the population level (like real populations) is nonstationary. This means that long-run properties of the model are mostly undefined. Moreover, while in expectation N grows without bound, the opposite outcome is also possible: the population may become extinct. (In that case, $\lambda(0) = \mu(0) = 0$, and the economy terminates.) Finally, as is always the case in CARA economies, prices and dividends can become negative, so that price-scaled quantities can become infinite. In thinking about average properties of sample paths, then, we will always want to condition on the event that such outcomes are avoided.[7]

Now consider liquidity in this market. The situation facing an agent who needs to buy shares (i.e., a new arrival) is one of an upward-sloping supply curve. In fact, we can immediately compute this curve since current holders of shares have the same demand function as the arriving agent:

$$\text{supply} = Q(P) = X^{(1)} - \sum_{n=1}^{N-1} \alpha(P) = X^{(1)} - (N-1)\alpha(P).$$

Equivalently, current holders of shares will only part with Q of them at a price that increases with Q. This means the new agent will have a price impact that increases in the amount she wishes to trade. The impact is a real cost in the sense that she pays more for the exact same dividend stream than she would if her demand were less – and she takes this into account in forming her optimal portfolio.

Thus, even though markets are frictionless, they are not perfectly liquid. This is the notion of liquidity used by Pagano (1989) which he calls the "absorptive capacity" of the market. As in Kyle (1985) type models, this capacity can be quantified by the amount of the price impact for a given order quantity.[8] But it is important to recognize that, unlike those models, it is not determined by asymmetric information or a zero-profit constraint of an intermediary, but simply by the risk-bearing capacity of the entire economy.

Because markets are illiquid, net order flow moves prices in this model. From the third term in Eq. (2), this "price pressure" effect contributes an additional component of volatility beyond pure "fundamental" volatility, i.e., from dividend news. Even though the risk aversion of individual agents is constant, the aggregate risk discount is affected by the impact of the aggregate trade demands.

The precise expression for the price impact function is found by writing the inverse supply function as $P(Q)$ and differentiating with respect to Q. This yields the illiquidity measure

$$\mathscr{I} = \frac{\gamma\sigma_\Theta^2/r^2}{N-1}. \tag{3}$$

[7]Explicit illustrations of the population dynamics and the intensity functions are given in Section 3 below.

[8]The degree of illiquidity can also be quantified by the size of the (hypothetical) bid-ask spread that would obtain if nontrading agents were required to make two-sided markets for those who were entering and exiting. If the population is competitive, the resulting spread (per unit volume) will be proportional to the slope of the market supply curve.

This is closely related to the illiquidity measure proposed in Johnson (2006). In more general economies the price impact computation may need to take into account the effects of simultaneously altering agents' holdings of *two* assets: the risky security and whatever it is exchanged for. In the current model, agents trade assets for the consumption good, or, equivalently, units of the riskless asset (whose price, $P^{(0)}$, is unity). Given CARA preferences, holdings of this asset do not enter agents' demands or equilibrium prices. In the general case, given a representative agent, one computes the price impact function as the derivative[9] of the pricing function with respect to holdings of the risky asset, $X^{(1)}$, when holdings of the unit of exchange, $X^{(0)}$, are varied according to the value-neutral condition $\mathrm{d}X^{(0)}/\mathrm{d}X^{(1)} = -P^{(1)}/P^{(0)}$. Referring to Eq. (2), here that yields

$$\mathscr{I} = \frac{\gamma\sigma_{\Theta}^2/r^2}{N},$$

which is virtually the same as (3). This latter form would also apply in any representative agent formulation of the economy in which (2) holds, for example, with a single agent and time-varying supply or risk aversion. In the current setting, the theoretical justification for the measure can be seen explicitly via the actual trading decisions of (disaggregated) agents.

The concept of liquidity studied here abstracts from many factors—including information asymmetry, search costs, and the financial constraints of intermediaries—that certainly come into play in determining trading costs in the real world. An important question, which this paper will not address directly, is how large a fraction of the total this preference-based component is. One measurement strategy is to gauge the (permanent) price response to informationless demand shocks.[10] Overall, empirical estimates of these pure "price pressure" effects are often at least as large as trading cost measures of illiquidity. From another angle, Johnson (2006) presents evidence that, in realistically calibrated equilibrium models, the implied level and dynamics of $\mathscr{I}$ are consistent with empirical measures of the illiquidity of the aggregate market. In other words, this source may account for a large fraction (perhaps even all) of the systematic component of liquidity. These arguments suggest that the preference-based component of illiquidity is significant, and that, at least at a high enough level of aggregation, one may be justified in not modelling the other components. Conversely, to the extent that the present paper succeeds in capturing a new dimension of liquidity dynamics (namely, the relationship with volume), it constitutes additional evidence on the relevance of this concept.

From expression (3), illiquidity in this model has a particularly simple form that captures some basic intuitions. First, it increases with the level of risk. This is a well-documented empirical property, which is sometimes attributed to inventory risk of intermediaries. Here there are no intermediaries. Instead, *any* agent, if asked to accommodate trade demand, will do so more readily when the asset in question is safer.[11] Second, the level of risk aversion directly affects this willingness to substitute from risky to riskless assets. More specifically, what matters to someone who needs to trade is the aggregate willingness to undertake such substitution: hence the inverse dependence of $\mathscr{I}$ on N. (Note that illiquidity becomes infinite when N hits one. This makes sense: if you are the only person in the economy, there is no one else to sell to.)

The most important feature of $\mathscr{I}$ is that it is stochastic. As the population varies, so does the market's total capacity to accommodate risk transfer. Agents thus face liquidity risk. They will suffer actual wealth losses in the form of falling stock prices if liquidity declines. This is another well-supported empirical feature of liquidity. While other mechanisms (for example, stochastic volatility) could generate similar dynamics, I have picked a particularly simple one in order to make the relationship with the process of trade as transparent as possible.[12]

[9]In Johnson (2006), $\mathscr{I}$ is actually defined as the log derivative, i.e., in elasticity units. In an arithmetic CARA economy it is more natural to use the simple derivative.

[10]Such measures can be made from pre-announced share issuances (Loderer, Cooney, and Van Drunen, 1991), or IPO lock-ups (Field and Hanka, 2001), from index inclusions or reweightings (Shleifer, 1986), from NYSE listings (Kadlec and McConnell, 1994), or from predictable mutual fund flows (Coval and Stafford, 2005).

[11]Note that it is the share price risk $\sigma_P = \sigma_\Theta/r$ that matters. This accounts for the apparent inverse dependence of $\mathscr{I}$ on r.

[12]Appendix B presents further evidence on the joint dynamics of prices, liquidity, and volatility in this model.

2.3. Volume and liquidity

Having characterized volume and liquidity, we can now analyze the relationship between them. The main results are mostly intuitively straightforward, although analytically somewhat intricate. I state them here without proof, and verify them for a broad set of parameter values in Section 3. It will sometimes be convenient to state results in terms of liquidity, which I define as $\mathscr{L} = 1/\mathscr{I}$, rather than illiquidity.

First, consider the nonrelationship puzzle discussed in the introduction. This can be viewed from a number of perspectives in the context of the model, all of which come down to the fact that liquidity is driven by the population size N, whereas volume is not.

A first, obvious, result is that expected volume does not affect liquidity.

Implication 2.1. *Liquidity does not increase with the level of trading intensity.*

Trading intensity, or expected volume, is governed by the parameter μ_1 (which also determines $\lambda(N)$). This parameter does not affect the expression for $\mathscr{I}$. While this is only a comparative static conclusion, the same logic would apply if the model were extended to allow μ_1 to be time-varying. So the model implies, for example, that seasonal reductions in trading activity, if they affect liquidity at all, do so only through a reduced rate of fundamental risk, σ_Θ.

Next consider the contemporaneous correlation between volume and liquidity innovations. Since expected volume in the model is constant, the question becomes whether volume is conditionally correlated with liquidity changes. As in the studies cited in the introduction, the answer is negative.

Implication 2.2. *Volume innovations are approximately conditionally uncorrelated with liquidity innovations.*

This result obtains because volume responds symmetrically to arrivals and departures whereas liquidity responds antisymmetrically. Again using the notation of Proposition 2.1, liquidity innovations are proportional to $\int \mathrm{d}A_t - \int \mathrm{d}D_t$. So the cross product $\mathrm{E}_{t_1}[\Delta\mathscr{L}_{t_1,t_2}\mathscr{T}(t_1,t_2)]$ is

$$\mathrm{E}_{t_1}\left[\left(\int_{t_{n-1}}^{t_n} \mathrm{d}A_t - \int_{t_{n-1}}^{t_n} \mathrm{d}D_t\right)\left(\int_{t_{n-1}}^{t_n} \mathrm{d}A_t/N_t + \int_{t_{n-1}}^{t_n} \mathrm{d}D_t/N_t\right)\right],$$

which is effectively zero, since the D and A processes are very nearly identically distributed.

Finally, the model goes further and suggests that there should be no (long-run) relation between levels of volume and liquidity. This is consistent with the evidence from vector autoregressions that, beyond the weak innovation correlations, there is also little evidence of structural linkage from the VAR coefficients.

Implication 2.3. *The level of volume in any future interval and the level of liquidity at the start (or end) of that interval are approximately uncorrelated, given today's information.*

Again, this is immediate from the fact that volume is roughly *i.i.d.* and does not scale with N, the population size, whereas liquidity does. A larger population does mean more arrivals and departures; however, each agent trades a smaller fraction of total supply. More generally, the degree of rearrangement of the agents in the model will remain roughly constant no matter which direction the overall population evolves.

Together these implications explain why the intuition from microstructure models concerning liquidity determination vanish when one views liquidity as a property of equilibrium market demand. Periods of higher volume do not make agents any more willing to facilitate trade because doing so does not yield them any profit. Market clearing spreads the net impact of trade across all agents equally. The gross (unsigned) amount of such trade does not determine whether demand curves steepen or flatten.

Given this logic, it is worth asking whether the model can simultaneously be consistent with the fact that there *is* a positive relationship between turnover and liquidity cross-sectionally. There is no cross-section of assets in the model's economy. If there were, however, the model would run into a more fundamental problem: there would be no cross-section of turnover. With identical agents, each would always trade the same basket of stocks, namely, the market portfolio. Hence, all stocks would experience the same turnover (and the volume of each would be proportional to its shares outstanding). This is, of course, a difficulty for many asset pricing models and points to the need for some additional heterogeneity.

On the other hand, with multiple assets there would be a cross-section of liquidity. The natural generalization of Proposition 2.3 would lead to a CAPM-like pricing result, and $\mathscr{I}$ would again be proportional to the risk discount, i.e.,

$$\mathscr{I}^{(j)} = \frac{\gamma \beta^{(j,M)} \sigma_M^2}{N-1},$$

with M denoting the market portfollio. This expression shows that a negative association between *volume* and illiquidity could be automatically recovered if systematic risk (beta) declines with the number of shares outstanding. Going further, it suggests that a negative association between *turnover* and illiquidity might be possible if turnover were negatively related to risk. For example, if higher beta stocks *also* were subject to more costly information acquisition, leading to less frequent trade, then high $\mathscr{I}^{(j)}$ would be associated with lower turnover (as well as volume). At the same time, the no-relationship results above would continue to apply dynamically for each asset.

2.4. *Volume and liquidity risk*

The implications derived above do not mean that there is *no* relationship between volume and liquidity in the model. In fact, the model implies that there should be a direct connection between volume and *the scale* of changes in $\mathscr{I}$.

This association should also hold on a number of levels. Most directly, higher expected volume means higher liquidity risk.

Implication 2.4. *Liquidity risk does increase with the level of trading intensity.*

Expected volume scales with μ_1. But as trading intensity rises, the variance of both arrival and departure processes increases, leading to more extreme population changes—in either direction.[13] Liquidity risk—the variance of $\mathrm{d}\mathscr{I}/\mathscr{I}$—coincides with the variance of $\mathrm{d}N/N$. The properties of Poisson processes would then suggest that liquidity variance scales *linearly* with μ_1, and hence with expected volume.

A more surprising implication of the model is that, even holding expected volume constant, higher realized volume should be associated with higher contemporaneous liquidity risk. That is, the joint distribution of volume and (log) liquidity changes will be cone-shaped, with the conditional variance of the latter rising with the former.

Implication 2.5. *The scale of liquidity innovations rises with realized volume.*

The intuition here is important as it constitutes the core of the model's description of the joint determination of volume and liquidity. If volume is very low then *both* arrivals and departures must have been low. So liquidity cannot have changed much. Likewise, if volume is very high, then *either* arrivals or departures must have been large. But they are independent of each other, so it is unlikely both were large. Hence $|\Delta N| = |A - D|$ is likely to be large, which entails a large change in $\mathscr{I}$.

In the model, then, both expected and unexpected (contemporaneous) volume contain information about the second moments of liquidity changes. When confronting the data, it will be of interest to also examine the properties of liquidity risk when conditioning on *both* expected and realized volume. This raises the somewhat subtle question of whether the strength of the (positive) relation between realized volume and liquidity variance itself scales with expected volume. Equivalently, we want to know how the shape of the joint distribution is perturbed when expected volume changes.

We have seen that, unconditionally, liquidity variance scales linearly with expected volume, as does realized volume. A result that one might expect, then, is that the sensitivity of liquidity variance to realized volume is scale free.

[13]As with Implication 2.1 above, this is a comparative static result, which, again, could be expected to carry over to a generalized model with time-varying μ_1.

Implication 2.6. *When expected volume changes, the elasticity of liquidity risk with respect to realized volume is approximately unchanged.*

This is not a literal implication of the theory. But, as shown in Section 3, it turns out to hold to a reasonable approximation. Its practical importance is that it guides us empirically, suggesting that a constant elasticity description of liquidity variance is likely to suffer little from misspecification.

As a final implication, the model suggests that the variance of (log) liquidity changes should also scale with the level of illiquidity. To see this, note that population changes $A - D$ over the interval $\mathrm{d}t$ are (roughly) the difference between two independent Poisson draws each with variance $\approx \mu_1 N\,\mathrm{d}t$. So percent population changes, $\mathrm{d}N/N$, which are equivalent to percent liquidity changes, have variance like $\mu_1\,\mathrm{d}t/N \propto \mathscr{I}$. Hence:

Implication 2.7. *Liquidity risk varies as the square root of the level of illiquidity.*

This property implies that liquidity is negatively skewed: rising variance with falling N leads to a fatter left tail. Or, more simply, liquidity can collapse faster than it can expand. This asymmetric liquidity risk certainly accords with conventional wisdom, and will be examined in the empirical tests below.

To summarize, the model makes a number of detailed predictions about liquidity dynamics. Most importantly, it implies what is (as far as I know) a novel relation: the second moment of liquidity increases with volume. This prediction, if accurate, is interesting in its own right as it sheds light on the fundamental drivers of liquidity risk. At the same time, the prediction constitutes a testable implication of the underlying view of liquidity advanced here.

3. Numerical illustrations

This section establishes that the implications deduced above are, in fact, valid for a broad range of parameter values. The first step is to verify that it suffices to consider only a couple of the parameters to characterize solutions. Then simulations are used to establish the properties in question.

3.1. Model solutions and parameter dependence

In the model of Section 2, volume and liquidity dynamics are driven entirely by the arrival and departure processes, which are characterized by their respective intensity functions $\lambda(N)$ and $\mu(N)$. So in establishing the generality of dynamic properties, it suffices to vary only the parameters that affect these intensities.

The function $\mu(N)$ is linear and is characterized by its slope μ_1. Unfortunately, $\lambda(N)$ is determined by the solution to a second-order difference equation involving all the model parameters. As shown in the appendix, however, this equation (and its boundary conditions) only involve μ_1, r (the riskless rate), and two other coefficients, p and h_∞, which are nonlinear functions of the primitive parameters. These four dimensions can be further reduced to three by adopting the assumption that r is equal to the rate of time preference ρ.

I now explore the remaining dimensions and argue that the solution for $\lambda(N)$ is actually effectively determined by μ_1. Specifically, I solve the difference equation and graph the function

$$\frac{\lambda(N)/(N+1)}{\mu(N)/N} = \frac{1}{\mu_1}\frac{\lambda(N)}{N+1} \equiv \xi(N)$$

and observe that its sensitivity to the other parameters is negligible. In that case, the function $\xi(N)$, once derived, yields $\lambda(N)$ as $\mu_1(N+1)\xi(N)$.

The three panels of Fig. 1 present the results of varying r, then μ_1, and then p while holding the others fixed.[14] The first thing to note about the solutions is their rapid convergence to unity, in accordance with Proposition 2.3. For low integers, the curves fall, and the departure rate slightly exceeds the arrival rate, because a boundary condition of the system requires $\lambda(1) = 0$. (See the discussion in Appendix A for more on this condition.)

[14]For brevity, I present only graphs with one degree of variation. Further exploration confirms that the graphs are also virtually indistinguishable if the parameters are varied in tandem.

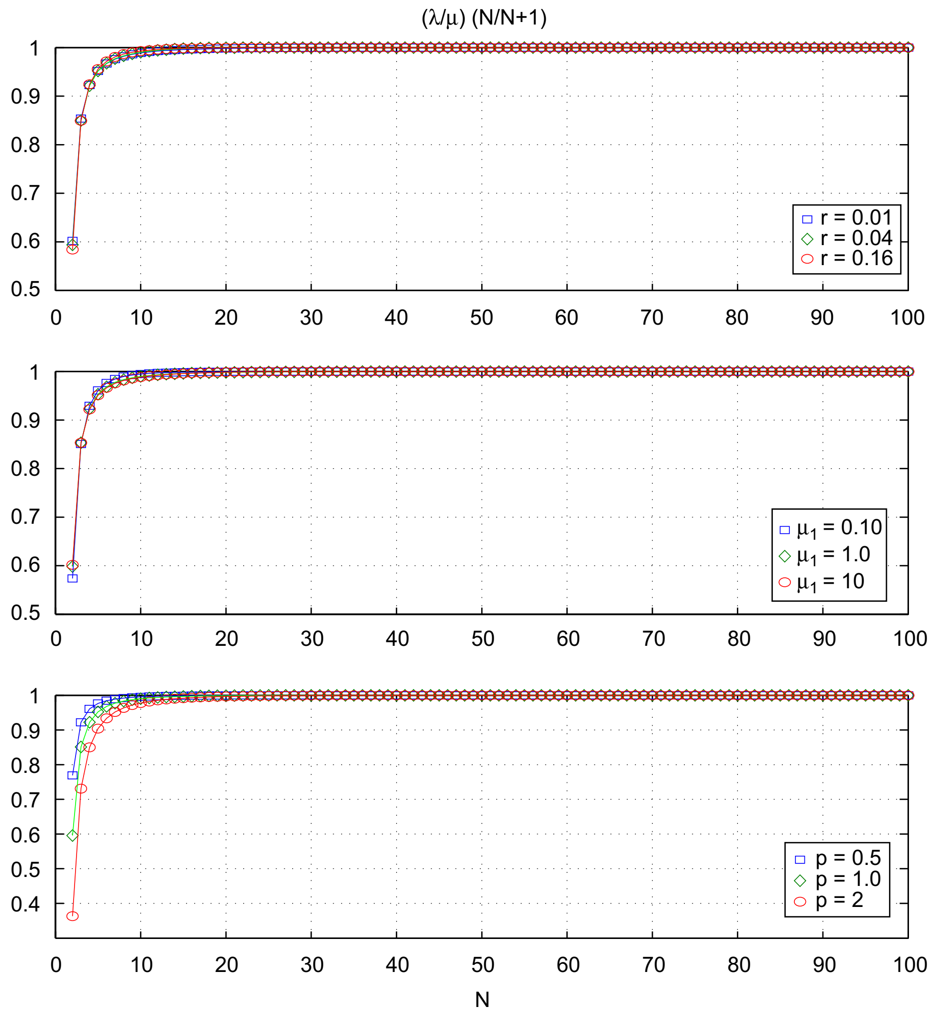

Fig. 1. Each panel plots the arrival intensity function as a (scaled) fraction of the departure intensity for different parameter values. The top panel fixes the parameters $\mu_1 = 1$ and $p = 1$, and varies the riskless rate. The middle panel fixes $r = 0.04$ and $p = 1$, and varies the per-agent departure intensity. The bottom panel fixes $r = 0.04$ and $\mu_1 = 1$, and varies the composite parameter p defined in Appendix A. For each graph, the rate of time preference is fixed at the riskless rate.

For all the graphs, however, it seems safe to assert that the parameter variation is minor. The different cases plot almost on top of one another. The bottom panel shows the most deviation. But, while visually detectible, the difference between these curves is still trivial. The intensity functions differ by less than 1% for integers greater than roughly four. In terms of the stochastic processes they parameterize, the functions are virtually indistinguishable. That is, one would not be able to discriminate between them statistically without extremely large samples of small populations.

The range of parameter values considered in this last panel is dictated by numerical considerations. It turns out that the system of equations one must solve to characterize the equilibrium is badly behaved for values of p much larger than those in the range shown. I show in Appendix B that this range is compatible with realistic calibrations of the model.[15]

[15]Economically, $p \equiv r(\gamma\ X^{(1)}\sigma_P)^2$ is equal to twice the percentage increase in the value function when the population goes from $N = 1$ to ∞. Bounding this quantity, in effect, limits the degree to which agents care about the number of other agents.

Returning to the plots, the conclusion that can be drawn is that, in studying the relationship between volume and liquidity, it suffices to vary the parameter μ_1 to examine virtually the entire parameter space. Besides this, however, we must recall that the economy in the model is nonstationary. This means that all statements about moments are inherently conditional on the current level of the population, N_0. (The unconditional correlation between volume and liquidity innovations, for example, is not defined. The one-week-ahead correlation, given $N = 100$ today, is however.) Hence, the current population (and, implicitly, the time horizon) are also effectively parameters that must be varied in assessing the generality of the model's properties.

3.2. Liquidity and volume

Table 1 shows the numerical expected values of both volume and population growth, $\log(N_T/N_0)$, for daily, weekly, and monthly observation intervals. The values are sample means over 100,000 replications of the population process for a wide range of choices of the starting population and the trading intensity. The continuous time processes are simulated at high frequency (2,000 steps per year) and then cumulated to the observation interval. For none of the simulations did the population hit zero.

The table helps illustrate some of the basic properties of the model already discussed. Volume (or turnover) scales linearly with trading intensity. In annualized percentage units, turnover is very close to 200 times μ_1. There is no significant change in expected volume with observation interval or with the level N_0. Both properties are consistent with turnover being *i.i.d.* Expected (log) population changes are very close to zero, confirming that the process N_t, and hence liquidity, are close to being martingales. The only exception to this statement occurs for small values of N_0, when, as seen above, the arrival intensity is less than the departure intensity and the population therefore tends to drift down.

Implication 2.1 asserts that liquidity does not change with the level of trading intensity. While that statement follows immediately from the expression for $\mathscr{I}$ and so requires no verification, the table also suggests a further statement, that expected liquidity *changes* do not increase with expected volume. This is still intuitive from the fact that the distribution of liquidity changes is symmetric about zero, whatever the value of μ_1. To the extent that there is any association in the table between the two first moments, it actually goes the "wrong" way. The drift downward in N (which corresponds to decreases in liquidity) for low N is higher when expected volume is higher.

The lack of an association between unexpected (or realized) volume and liquidity innovations is asserted by Implication 2.2. The first panel of Table 2 shows the correlations between these two quantities in the

Table 1
First moments

N_0	μ_1	Observation period					
		Daily		Weekly		Monthly	
		Turnover	Pop chg	Turnover	Pop chg	Turnover	Pop chg
10	0.10	21.02	−0.32	20.68	−0.45	20.96	−0.21
	1.00	206.1	−1.08	204.3	−1.95	209.0	−1.76
	10.0	2096	−18.46	2070	−16.74	2081	−19.79
100	0.10	19.60	0.06	19.90	0.11	20.06	0.00
	1.00	210.4	−0.30	198.6	−0.11	200.4	−0.34
	10.0	2009	−0.74	1989	0.59	2001	0.11
1000	0.10	19.96	0.01	19.84	0.01	19.95	0.00
	1.00	200.4	0.03	198.3	−0.09	199.4	0.02
	10.0	2001	0.13	1981	−0.08	1993	0.05

The table shows sample first moments for turnover and population change from 100,000 sample paths of the model of Section 2. Both numbers are reported as annualized percentages. The underlying continuous time model is approximated at a frequency of 2,000 steps per year. Turnover is computed under the assumption of continuous trade.

Table 2
Volume correlations

Trade frequency		Daily obs		Weekly obs		Monthly obs	
N_0	μ_1	Continuous	Daily	Continuous	Daily	Continuous	Daily
Panel A: *Correlation with liquidity innovations*							
10	0.10	0.02	−0.02	0.02	−0.03	−0.01	−0.04
	1.00	−0.02	−0.06	−0.01	−0.06	−0.01	−0.06
	10.0	0.00	−0.05	−0.01	−0.09	0.03	−0.15
100	0.10	−0.02	0.02	−0.01	−0.01	0.00	−0.01
	1.00	0.00	−0.01	0.00	−0.01	0.00	−0.02
	10.0	0.01	0.00	0.00	−0.03	0.00	−0.10
1000	0.10	0.00	0.00	0.01	0.01	0.00	0.00
	1.00	−0.01	−0.01	0.01	0.00	0.00	−0.01
	10.0	0.00	−0.01	0.01	0.00	0.00	−0.04
Panel B: *Correlation with squared liquidity innovations*							
10	0.10	0.93	0.93	0.94	0.95	0.85	0.85
	1.00	0.86	0.88	0.73	0.76	0.46	0.48
	10.0	0.59	0.70	0.31	0.41	0.15	0.24
100	0.10	0.88	0.90	0.75	0.77	0.48	0.50
	1.00	0.61	0.75	0.34	0.43	0.17	0.21
	10.0	0.25	0.56	0.12	0.28	0.06	0.15
1000	0.10	0.61	0.76	0.34	0.43	0.17	0.21
	1.00	0.24	0.58	0.12	0.29	0.05	0.14
	10.0	0.08	0.45	0.04	0.23	0.02	0.12

Panel A shows sample correlations between volume and liquidity innovations from 100,000 sample paths of the model of Section 2. Panel B shows the correlations between volume and squared liquidity innovations for the same simulations. Correlations are shown under both continuous and daily trading.

simulation. The values are shown for both continuous trading and daily trading, i.e., using the respective expressions in Propositions 2.1 and 2.2 from Section 2. As expected, the correlations are close to zero over virtually the entire range of parameters. If anything, there is again a small *negative* relation under discrete trading when μ_1 is high relative to N. This happens because daily trading lowers total turnover relative to continuous trade by an amount that is proportional the number of transactions.

Implication 2.3 says that *levels* of liquidity and turnover are also uncorrelated. To illustrate, 100,000 model paths are simulated out to two years. Every week, the correlation between the liquidity level and the (next-week) turnover is computed. Fig. 2 shows the resulting correlations as a function of horizon for the case $\mu_1 = 10$, with the date-zero population set at $N_0 = 100$. The simulations show that the assertion is borne out, assuming continuous trade. With daily trade netting, the mechanism just described induces a negative correlation, which grows over time as the population dispersion increases. Under both trading arrangements there is clearly no positive long-run association.

Turning to second moments, Table 3 shows liquidity risk as a function of the sample path parameters. From the table, one can easily discern the square-root scaling with μ_1 and the inverse square-root scaling with N_0. This verifies Implications 2.4 and 2.7 and, in particular, illustrates that liquidity risk rises with expected volume (i.e., with μ_1). The table also shows the third moment of liquidity, confirming the prediction of negative skewness. This effect is small unless the probability of a population collapse (e.g., to $N = 1$ or 2) is appreciable.

The positive association between liquidity risk and contemporaneous or unexpected volume (asserted in Implication 2.5) is illustrated in the second panel of Table 2. This shows the correlations between volume and squared liquidity innovations. These are all positive and range as high as 95%. The association is especially strong when the transaction intensity is relatively low, suggesting that it would be easier to discern in daily

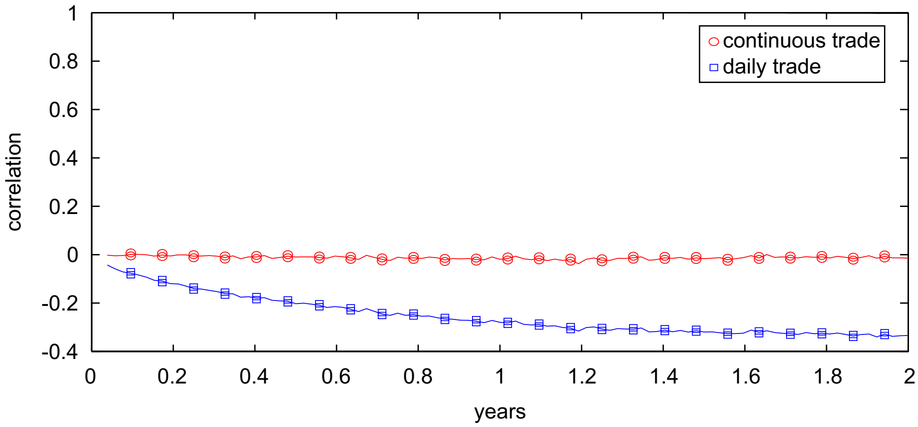

Fig. 2. The figure plots the correlations between levels of volume and levels of liquidity as a function of time horizon over 100,000 simulated paths. The paths set $\mu_1 = 10$ and $N_0 = 100$, and assume weekly observation intervals. The simulations are performed under the assumption of continuous or daily trade. The boxes and circles for each curve are shown at a width of ± 2 times the numerical standard error of the correlations.

Table 3
Liquidity risk

N_0	μ_1	Daily obs		Weekly obs		Monthly obs	
		Volatility	Skewness	Volatility	Skewness	Volatility	Skewness
10	0.10	13.74	−0.803	14.24	−0.737	14.48	−0.290
	1.00	42.52	−0.471	44.72	−0.272	46.05	−0.334
	10.0	135.23	−0.287	143.48	−0.398	152.72	−0.895
100	0.10	4.15	−0.031	4.41	−0.032	4.47	−0.024
	1.00	13.30	−0.025	13.93	−0.019	14.04	−0.076
	10.0	41.89	−0.051	44.11	−0.090	44.62	−0.191
1000	0.10	1.33	−0.017	1.39	0.001	1.40	−0.015
	1.00	4.19	−0.009	4.38	−0.012	4.45	0.003
	10.0	13.20	−0.008	13.89	−0.020	14.06	−0.043

The table shows sample standard deviation and skewness of liquidity innovations from 100,000 sample paths of the model of Section 2. Volatilities are shown as annualized percentages.

data or in quiet times. The assumption of daily trade netting strengthens the association somewhat relative to continuous trade.

While the numbers in the table make the basic point about volume-driven heteroskedasticity, the effect is perhaps better illustrated by the shape of the joint distribution. The scatter plot in Fig. 3 shows the distinctive cone- (or heart-) like shape that corresponds to having a positive correlation between squared liquidity innovations and volume. The case shown has $\mu_1 = 10$, $N_0 =$1,000 and assumes daily trade and daily observation. But the pattern is quite general and will form the basis of the empirical tests in Section 4.3.

The simulations can also address the question of the scaling of the joint dependence with the level of the parameters. Visually, the quantity of interest is the steepness of the slope of the cone surrounding the joint distribution. Mathematically, this corresponds to the elasticity of (conditional) variance with respect to volume. Table 4 estimates this quantity from the simulations. The table reports the log difference in variance estimates moving from the first to the fifth volume quintile, divided by the corresponding change in log volume. (This method breaks down for small trade intensities because the variables in question can only assume a few possible values. This accounts for the smaller range of parameters shown in the table.) The table indicates that this quantity is, indeed, stable, as suggested by Implication 2.6, for continuous trade. In fact, its

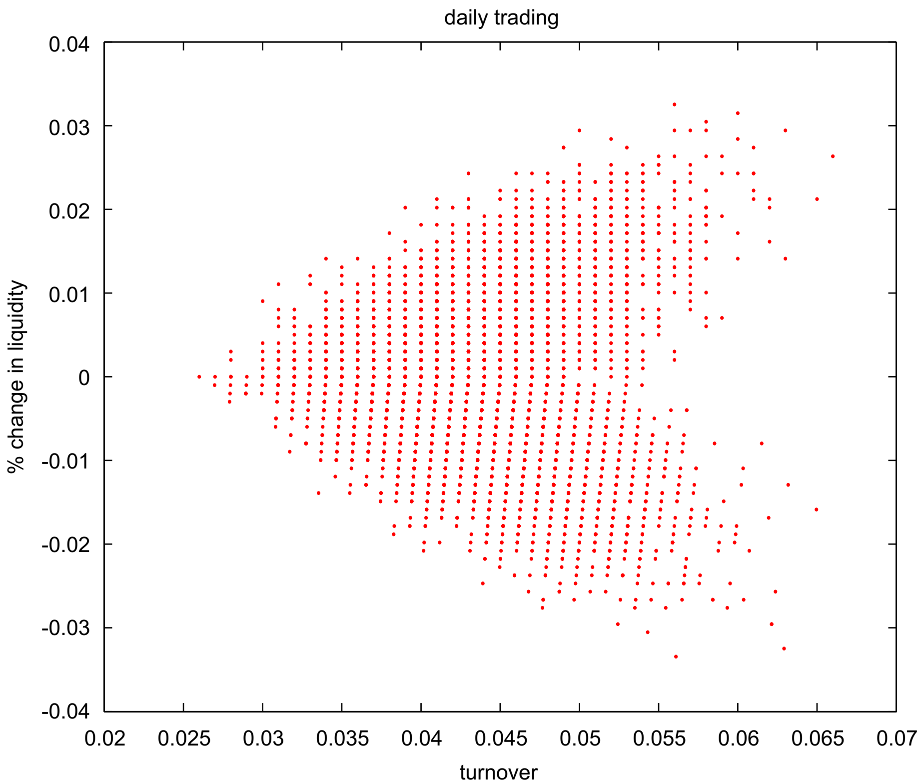

Fig. 3. The figure plots joint distribution of volume and liquidity changes using the simulations of Section 3. The paths set $\mu_1 = 10$ and N_0 =1,000, and assume daily trade and daily data.

Table 4
Elasticity of variance

Trade freq		Weekly obs		Monthly obs	
N_0	μ_1	Continuous	Daily	Continuous	Daily
100	0.10	1.14	1.21	1.20	1.33
	1.00	1.00	1.28	0.93	1.43
	10.0	1.00	2.36	1.02	2.47
1000	0.10	1.01	1.29	0.99	1.38
	1.00	1.06	2.14	0.95	2.44
	10.0	0.94	5.61	0.60	5.56

The table reports the log difference in liquidity variance estimates moving from the first to the fifth volume quintile, divided by the corresponding change in log volume for the simulated samples of Section 3.

value is near unity over a very wide range of values. With daily trade, the values are somewhat higher overall, and do tend to increase as trading intensity and the population size increase. Little variation results from increasing the observation interval from weekly to daily.

The numerical experiments in this section have focused on the relation between volume and liquidity, which is the paper's subject. In Appendix B the analysis is extended to consider several other dynamic interactions in the economy. A number of the additional predictions of the model seem to accord well with empirical findings, and offer potentially fruitful avenues for further research. At present, however, the task is to see whether the theory can successfully describe the association between volume and liquidity risk.

4. Empirical evidence

This section presents empirical evidence on the second-moment predictions of the model. The study uses data from the U.S. government bond and stock markets, which differ markedly along several dimensions. Treasury bonds are traded in a broker-dealer market and the data used are at weekly frequency. The stock data are monthly and the underlying exchange is a specialist market. The two types of securities are also governed by different regulations and obviously have very different cash flow characteristics (and, needless to say, neither is described by a perpetual arithmetic Brownian motion dividend stream).

Clearly a more accurate model of the assets and the institutional settings would be better capable of a comprehensive depiction of trade dynamics in either market. Here, however, the goal is limited to testing for a specific relationship, based on the theory's unique implications. By confronting the stylized and oversimplified model of Section 2 with diverse and complex data sets, the tests provide a demanding trial for the underlying theory.

4.1. Data

The tests below use aggregate time-series data for both markets. I now describe the construction of the main variables, discuss the samples, and present some summary statistics.

4.1.1. Turnover

The primary independent variable in the model is market volume. In constructing empirical proxies, we immediately face two issues stemming from the theory's simplifications. First, expected volume is not constant, so volume is not *i.i.d.* Second, the total quantity of securities is not fixed, as the model assumes.

The latter problem is relatively easily handled. In the model, if the quantity of shares changes by a factor, k, then all agents' holdings and trades would change by the same factor. Hence, scaling volume by supply—i.e., using turnover—should preserve all the predictions.[16] As an empirical matter, this requires some additional choices, to be discussed below.

With regard to the persistence of volume, I suggested in Section 2 that the theory could be extended to incorporate time-varying expected volume by allowing μ_1 to vary. If this intensity followed a (deterministic or stochastic) trend, this would induce autocorrelation in volume. As long as the required relationship between $\lambda(N)$ and $\mu(N)$ were preserved, the model's dynamic predictions could be expected to survive the generalization. Empirically, the tests will require us to separate out the expected component of turnover.

Turnover in the U.S. stock market can be measured fairly directly. I use the composite data series available from the New York Stock Exchange (NYSE), which give monthly market capitalization and dollar volume going back to 1934. I define turnover as the ratio of dollar volume to the average of beginning-of-month and end-of-month capitalization.

I estimate expected turnover in three steps. First, I subtract a deterministic trend from the raw series (in logs) using a Hodrick and Prescott (1980) filter. (Results are almost identical using polynomial or rolling-average trends.) Second, I compute the seasonal average over the detrended observations for each calendar month separately. Third, I compute the fitted value from an AR(1) model of the deseasonalized, detrended series. (There is no significant higher-order autoregressive dependence in the residuals.) Expected turnover for each month—as of the start of that month—is thus the sum of the trend and seasonal terms, plus the conditional expectation from the AR(1) regression.

For the government bond markets, things are somewhat less straightforward. I measure weekly volume from the New York Federal Reserve's survey of the positions and activity of primary dealers. This series goes back to 1996, and it contains separate volume numbers for primary dealer trades with interdealer brokers and with customers.[17] I focus on the total customer volume since the model depicts the trades of the individual agents who ultimately bear the risk of the assets, and not trades driven by the sharing of end-user demand

[16] Johnson (2006) establishes consistency properties of the measure $\mathscr{I}$ when the economy is extended to include stochastic supply.

[17] The breakdown reported is actually between trade with brokers and trade with nonbrokers. But since there is almost no direct dealer-to-dealer trade, the latter number must coincide closely with customer trade.

between liquidity providers. The customer volume series does omit customer volume not intermediated by primary dealers.[18]

To compute turnover, I scale volume by the total Federal debt held by the public. This series, from the Federal Reserve Bank of St. Louis, is reported quarterly. I use cubic splines to interpolate weekly supply numbers. There are a number of possible alternative scalings, representing different views of what should correspond to the outstanding supply of "the asset" in the model. The empirical conclusions below turn out to be robust to alternatives such as the outstanding amount of on-the-run issues.

To estimate expected volume, I then follow essentially the same steps as with the stock data. Because supply changes in the Treasury market are fully anticipated, I apply the procedure to volume, and then standardize expected volume by supply to get expected turnover. The seasonal component of volume is estimated at daily intervals (since weeks end on different days from year to year) via a centered average over the detrended observations within a one-week window (i.e., two trading days on either side). There is, again, no indication of autocorrelation in the residuals after subtracting a first-order autoregressive term. The procedure here (and for the stock data) estimates expected log turnover, whereas the model will need the log of expected turnover. I estimate the latter from the former by adding half the standard deviation of the estimated variance of the innovation residuals.

Fig. 4 plots the turnover series for both markets. The plots show both actual log turnover and the estimated series of beginning-of-period expectations (the dashed line). The lines are hard to distinguish because the level of turnover is highly predictable. The units are logs of per-period turnover ratio. So, for example, -1.60, the mean value for bonds, corresponds to roughly 1,000% per year (5,200 times $e^{-1.6}$). The stock mean of -3.7 corresponds to 30% per year, although recent levels have approached 100% (1,200 times $e^{-2.5}$). Note that the tests below use subperiods of the date ranges shown in the plot corresponding to the availability of the liquidity measures.

4.1.2. Liquidity

Unlike volume, there is no single consensus measure of the aggregate liquidity of a market. However, recent research has made several useful proxies available.

For the Treasury bond market, Fleming (2003) computes several liquidity measures (and associated trade statistics) from high frequency intradealer broker trade data in the GovPX data set.[19] The data are aggregated to the weekly level over the sample period 12/30/96 to 3/31/00. The data set tracks the current on-the-run issues for maturities of three months, six months, one year, two years, five years, and ten years.

This study uses two measures of bond illiquidity: bid-ask spread and price impact. The latter measure is most similar to the theoretical quantity $\mathscr{I}$ of the model. It is the coefficient from an (intraweekly) regression of price changes (or discount rate changes for the Treasury bills) on the net number of purchases, both observed at five-minute intervals. The bid-ask spread is the intraweek average spread, also in price or discount rate units. This measure provides a natural and intuitive alternative proxy. Moreover, the theory implies that $\mathscr{I}$ is directly proportional to this spread.[20] After converting all units to basis points, I aggregate each liquidity measure by averaging (in logs) across issues to get a composite market estimate.

For the stock market, I also use two measures of illiquidity, although neither is as direct as the bond market variables.

First, I employ the series of Pástor and Stambaugh (2003), which is based on the response of individual firm returns to their own lagged volume (multiplied by the sign of the current return). The idea is to capture the tendency of volume to induce return reversals via temporary price pressure. The response coefficient is measured by monthly regressions for each NYSE/AMEX listed firm. The equal-weighted average across firms

[18]The N.Y. Fed primary dealer volume series are measured on a Wednesday-to-Wednesday basis, whereas the liquidity series described below are measured Friday-to-Friday. When matching the series, I use the volume of the prior Wednesday-to-Wednesday (i.e., two days behind) as my independent variable. This should make it harder to detect volume-driven heteroskedasticity since it lowers the amount of contemporaneous information in the volume variable.

[19]I am very grateful to Michael Fleming for making his measures available.

[20]See note 8 in Section 2.

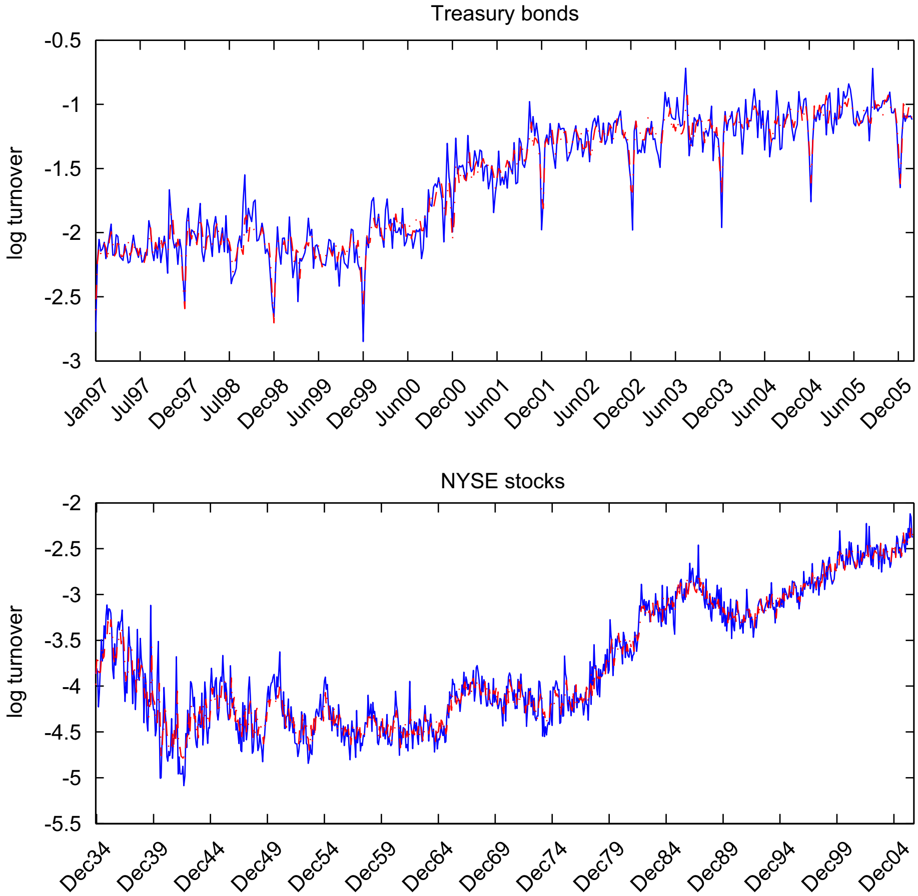

Fig. 4. Log annualized turnover is plotted for U.S. government bond and stock markets. The bond market series is the ratio of weekly nonbroker trades by primary dealers to total federal debt held by the public. The stock market series is the ratio of money volume of shares traded each month to the average of beginning- and end-of-month market capitalization of NYSE stocks. Each plot includes the expected turnover series (dashed line) described in the text.

is then weighted by the level of the market at the start of the month, giving their aggregate liquidity level. (The series is available from The Wharton Research Data Service.) Data run from February 1962 through December 2005.

The idea of volume-induced reversals behind the measure suggests that the response coefficient should be negative, and thus multiplying it by negative one should yield a measure of an implicit cost, or illiquidity rather than liquidity. However, the sign of the measure is not literally restricted, and it switches often in the data, making the cost interpretation problematic. For purposes of this study, in specifications using the log of trading cost I simply use the level of the series.

The second illiquidity series comes from the latent common factor model of Hasbrouck (2006). The underlying model in that paper identifies effective trading cost via the degree of negative autocorrelation in stock returns (as from a "bid-ask bounce"). The magnitude of the cost together with the difference between the sign of the last trade-direction indicator on successive dates determines the negatively autocorrelated (non-fundamental) component of recorded returns. Temporal variation in the cost term is assumed to follow a single-factor specification. The coefficients in this specification, the factor realizations, and the trade indicators are all treated as unobservable latent variables and are jointly estimated, year by year, for all U.S. stocks via a Gibbs sampling algorithm. (The estimates are available on Joel Hasbrouck's web site.) I then form a monthly time series from the product of each month's factor realization with the weighted average cost coefficients using all NYSE stocks for the corresponding year. This yields a time series spanning the months January 1926 to December 2005. Unlike the Pástor and Stambaugh (2003) measure, this series is restricted to positive values, and is directly interpretable as a percentage cost.

Fig. 5 shows the illiquidity measures for both markets. The two Treasury series are clearly picking up very similar variation, which accords with the model's view that one should be proportional to the other. The units of both are logs of basis points (per trade). So a value of -1 corresponds to less than half a basis point.

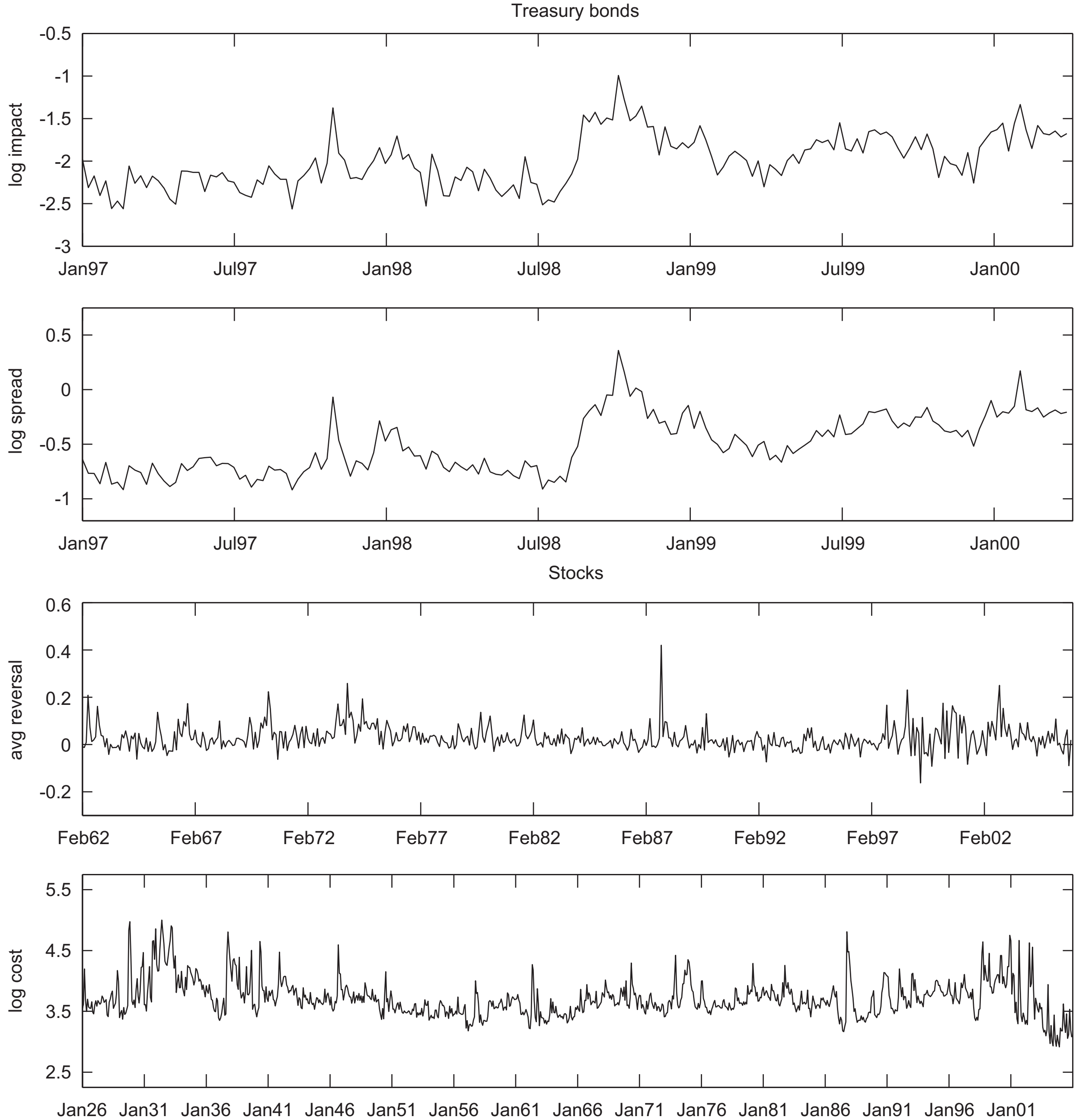

Fig. 5. It shows the four measures of illiquidity used in the empirical tests. The top two panels show the average across on-the-run Treasury securities of the log price impact coefficient and log bid-ask spread as reported in Fleming (2003). The third panel shows the level of the liquidity measure of Pástor and Stambaugh (2003) times negative one. The fourth panel shows the latent common factor innovations computed from the data of Hasbrouck (2006). Units for the first, second, and fourth panels are log basis points of trading cost.

Table 5
Summary statistics

	Log Turnover				Illiquidity changes				Returns		Volatility changes	
	ES	US	EB	UB	S1	S2	B1	B2	S	B	S	B
Mean	−3.747	−0.013	−1.591	−0.007	0.000	−0.001	0.002	0.003	0.009	0.000	−0.001	−0.001
Std dev	0.677	0.159	0.448	0.116	0.066	0.215	0.204	0.127	0.0045	0.00025	0.366	0.351
Skewness	0.494	0.746	−0.188	0.182	0.048	0.902	0.431	0.496	−0.859	0.090	0.309	−0.276
ρ_1	0.981	0.018	0.956	−0.030	−0.512	−0.149	−0.365	−0.268	0.011	0.102	−0.348	−0.322
ℓ_3	0.00	0.00	0.00	0.54	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.03
N obs	862	862	477	477	526	959	169	169	854	169	854	169

The table shows sample statistics for the variables described in Section 4.1. Stock market data are monthly and government bond data are weekly. The first four columns give values for expected stock turnover (ES), unexpected stock turnover (US), expected bond turnover (EB), and unexpected bond turnover. Turnover is expressed as the log of the ratio of per-period volume to supply. Unexpected turnover is the log of turnover divided by its expectation. Columns four to eight give statistics for changes in illiquidity measures. S1 refers to the Pástor and Stambaugh (2003) proxy (times negative one); S2 is the log of the Hasbrouck (2006) latent common factor measure. B1 and B2 are logs of average price impact coefficients and bid-ask spreads, respectively, both from Fleming (2003). The ninth and tenth columns are for benchmark returns in each market: S is the NYSE composite index, and B is the return on the on-the-run three-month T-bill. Returns are continuously compounded. The last two columns show the statistics for the percent changes in the volatility of these two series. Stock volatility in each month is estimated with daily returns, and T-bill volatility each week is estimated with five-minute returns. The fourth line shows the first autocorrelation coefficient. The fifth line gives the p value for the Ljung and Box (1978) heteroskedasticity test using three lags of the squares of each series. The last row shows the number of observations from which the statistics are computed.

Stocks are more costly to trade. The log values from the Hasbrouck (2006) series in the bottom panel are in the range of four, or roughly 50 basis points.

4.1.3. *Summary statistics*

In addition to illiquidity and turnover measures, the empirical work also uses realized return and volatility estimates for the two underlying markets. For the stock market, the return is taken as the percent change in the NYSE composite average over each month, and the volatility is the standard deviation of the daily returns within that month. For the Treasury market, return and volatility series are based on the on-the-run three-month Treasury bill. Weekly returns are computed from the GovPX data, and volatility is measured as the standard deviation of the yield changes measured over five-minute intervals within each week. These data are again from Fleming (2003). Yield volatilities are converted to return volatilities via a standard duration transformation.

Table 5 shows summary statistics for returns, volatility, illiquidity, and turnover. Expected turnover in both markets is highly persistent. Unexpected turnover is highly volatile and positively skewed. Percent changes in all liquidity measures are negatively autocorrelated, reflecting the apparent stationarity of the levels seen in the plot above. Illiquidity changes are also highly volatile and positively skewed. (The exception is the Pástor and Stambaugh (2003) series, labeled S1, which is less skewed and much less volatile.) Note that, on an annualized basis, bond illiquidity volatility is higher than that of stocks, even though the level of illiquidity is higher for stocks (e.g., for S2 the annualized volatility is $0.215\sqrt{12}$ or 74%, versus $0.127\sqrt{52}$ or 92% for B2). All the series (except for unexpected bond turnover) show strong evidence of heteroskedasticity.

4.2. *Specification*

Based on the analysis in Section 2, the model's main implications about liquidity risk can be summarized by the expressions

$$\mathrm{Var}_t[\Delta \log \mathscr{I}_{t+1} | \mathscr{T}_{t+1}] \propto \mathscr{T}_{t+1}^{\delta_1} \, [\mathrm{E}_t(\mathscr{T}_{t+1})]^{\delta_2} \mathscr{I}_t^{\delta_3}.$$

In words, time-t liquidity risk scales with expected turnover and the level of illiquidity. Further, given contemporaneous turnover (but no other contemporaneous information), liquidity risk also scales with that quantity.

The most distinctive prediction of the theory is that $\delta_1 > 0$. This captures the economic insight that large volume is necessary for large liquidity changes of either sign, and that small volume is sufficient to ensure small liquidity changes.

The model also predicts that the sign on expected turnover, δ_2, should be positive. When trading intensity goes up, the population flux rises, leading to bigger changes in liquidity. When *also* conditioning on realized turnover, however, the sign becomes ambiguous because both realized turnover and liquidity risk scale with expected turnover. If we introduce scaled realized turnover,

$$\mathscr{U}_{t+1} \equiv \mathscr{T}_{t+1}/\mathrm{E}_t(\mathscr{T}_{t+1})$$

(redefining δ_1 to be its exponent), then the prediction that $\delta_2 > 0$ applies to the conditional specification as well.

Last, the prediction of positive skewness for illiquidity means that the parameter δ_3 should be positive. Taking the theory literally, in fact, we would expect $\delta_3 \approx 1$.

These considerations immediately suggest testing the model by estimating specifications of the form

$$\begin{aligned} &\Delta \log \mathscr{I}_t = X_t \beta + h_t^{1/2} \varepsilon_t, \\ &\log h_t = \delta_0 + \delta_1 \log \mathscr{U}_t + \delta_2 \log \bar{\mathscr{T}}_t + \delta_3 \log \mathscr{I}_{t-1}, \\ &\varepsilon \sim \mathrm{N}(0,1), \end{aligned} \tag{4}$$

where X_t is a vector of controls and $\bar{\mathscr{T}}_t$ is expected turnover. Section 4.3 below will fit such models via quasi-maximum likelihood.[21]

Before proceeding to the estimation, however, it is important to carefully consider the role of the control terms in the mean equation.

Ordinarily, one might think that any and all covariates of liquidity changes should be included. But that is not the case here. The reason is that the goal of the specification is to test for the hypothesized second-moment effects—which condition *only* on turnover. Conditioning on more information in the mean equation should, according to the model, weaken the association between volume and liquidity risk. To take the extreme case, the model suggests that *all* variation in liquidity could be explained by observing ΔN, which would leave no conditional volatility at all. More generally, including anything contemporaneously correlated (under the model) with population changes will absorb some of the liquidity variability and lessen the power of the tests to detect the conditional heteroskedasticity.

Hence, it is important to stress that the goal of the empirical work is *not* to come up with the best specification, or highest R^2, for liquidity changes. Rather, the predictions to be tested are about the distribution of these changes when we do *not* condition on ΔN. For this reason, the tests could actually be less informative if measures of order flow or price changes were included in the specification of the mean. This is not because they don't matter, but because, according to the model, they matter too much.

What about lagged predictors? According to the model, liquidity changes should be unpredictable. So there is no danger of absorbing endogenous variation by including extra lagged conditioning terms. Indeed, to the extent that liquidity changes *are* predictable (and the model is misspecified), including the predictors should sharpen the inference by correctly isolating the variance of the true residual.

A natural candidate, then, would be to include lagged liquidity changes. Although the model thinks liquidity is nonstationary, in the samples introduced above liquidity levels appear stationary and changes are negatively autocorrelated (see Table 5). While it certainly seems likely that liquidity levels have declined over the long run, if the series truly are stationary, then including lags in the specification is

[21] The estimation is "quasi" in the sense that the model does not literally imply normality of the error terms or log-linearity of the variance. The results of small-sample experiments (available upon request) with calibrated data suggest that the procedure shows little bias and, in combination with numerical standard errors computed from the outer product of the scores, yields appropriate confidence intervals.

advisable.[22] The danger is that spuriously including these lags (if stationarity is false) would distort inferences about the variance. Specifically, a large increase followed by a large decrease could (incorrectly) be attributed to negative autocorrelation, when both were due to large volume innovations.

Another possible source of misspecification comes from time-varying volatility of returns. The model takes σ_Θ to be constant, while, in practice, there is a great deal of variation in the volatility of every financial time series. At least from a comparative static perspective, the model implies that changes in fundamental volatility, σ_Θ, have no effect on trade since these would affect all agents equally. However, from Eq. (3), illiquidity scales directly with σ_P^2. Intuitively, even in more general economies, anything that causes asset risk to rise will steepen demand curves. This implies that controlling for volatility changes may be important in explaining liquidity changes.

Here, again, care is needed because *realized* price volatility is to some degree endogenous. It depends on illiquidity since illiquidity determines the price impact of random flow shocks. This effect has nothing to do with stochastic volatility of fundamental news. Even if σ_Θ were constant, realized volatility in every period would be correlated with the illiquidity level because the latter dictates the magnitude of the price response to net trade demand. Including changes in realized price volatility in the conditional expectation specification could then weaken the ability to detect the second-moment effects. The degree of endogeneity of volatility depends on the size of the flow-driven (or price pressure) component of price changes relative to the news-driven component. If the former term is small, controlling for time-varying volatility should correct a potential misspecification in the model without affecting the predicted variance relationships in the residual.

4.3. Results

Table 6 shows point estimates for the parameters in the variance specification estimated individually and then together. Overall, the evidence strongly favors the model's predictions. For all four series, the estimates of δ_1 are positive and highly significant. The magnitudes range from 0.509 to 2.647, which are economically significant as well. For example, a value of two means that every $x\%$ increase in unexpected turnover translates into an equivalent percentage increase in liquidity standard deviation. From Table 5, the standard deviation of unexpected turnover is roughly the same size as that of liquidity, suggesting that the association here may account for a large fraction of liquidity variability.

Verifying the contemporaneous correlation between volume and liquidity risk is the key empirical result, since this relationship is necessary if changes in risk-bearing capacity (or participation) do play a role in liquidity determination. However, the other predictions of the model also find substantial support in the table.

For three of the series, there is evidence of a strong linkage between changes in expected turnover and liquidity risk. (The effect is not significant for the bid-ask measure in the bond market.) The magnitudes of the coefficients are similar to those for unexpected turnover. Unlike the previous finding, this is a predictive effect: liquidity risk rises *ex ante* when trading intensity is high. Moreover, expected turnover is highly persistent, suggesting that this relationship may be important for understanding longer-term trends in liquidity risk. It is noteworthy, for example, that the large increases in stock trading volume in the recent past have coincided with highly uncertain times in terms of liquidity. (This is visually evident in Fig. 5.)

Finally, the own-elasticity of variance effect captured by δ_3 is also present. The effect ranges from extremely large (for the Pástor and Stambaugh, 2003 proxy) to insignificant (for the bond price impact measure). Note that the former series has not been subjected to a logarithmic transformation before differencing. So its innovations will naturally be more positively skewed, which is what this coefficient measures.

The estimates in the table are all obtained under the assumption of a constant conditional expectation for liquidity innovations, as implied by the model. Table 7 verifies that the general conclusions are robust to inclusion of further covariates in the mean specification. Most importantly, δ_1 maintains its magnitude and significance for all series and all sets of controls.

The first panel controls for changes in volatility and for lagged changes in illiquidity. The only additional frailties that this specification reveals is that the conditional variance effect is now no longer statistically

[22] There is no easy generalization of the model that keeps liquidity stationary. If the population level is mean-reverting, construction of the equilibrium is much more complicated.

Table 6
Heteroskedasticity tests

Panel I: *Stock market illiquidity series*

	S1				S2			
	i	ii	iii	iv	i	ii	iii	iv
δ_1	0.509	—	—	1.457	0.744	—	—	2.319
	(0.054)			(0.255)	(0.034)			(0.123)
δ_2	—	0.470	—	0.511	—	0.660	—	0.681
		(0.051)		(0.061)		(0.033)		(0.034)
δ_3	—	—	4.305	4.831	—	—	0.697	1.210
			(0.305)	(0.457)			(0.068)	(0.090)

Panel II: *Bond market illiquidity series*

	B1				B2			
	i	ii	iii	iv	i	ii	iii	iv
δ_1	2.030	—	—	1.948	1.490	—	—	2.647
	(0.564)			(0.713)	(0.300)			(0.524)
δ_2	—	2.369	—	2.159	—	0.926	—	−0.072
		(1.012)		(1.080)		(0.560)		(0.809)
δ_3	—	—	0.233	0.052	—	—	1.118	1.019
			(0.384)	(0.488)			(0.354)	(0.430)

The table reports estimates for the illiquidity variance model for the U.S. government bond and stock market samples described in Section 4.1. The variance specification for percent changes in illiquidity is $\log h_t = \delta_0 + \delta_1 \log \mathscr{U}_t + \delta_2 \log \overline{\mathscr{T}}_t + \delta_3 \log \mathscr{I}_{t-1}$, where $\mathscr{U}$ is contemporaneous unexpected turnover, $\overline{\mathscr{T}}_t$ is expected turnover at time $t-1$, and $\log \mathscr{I}_{t-1}$ is lagged illiquidity. The conditional mean is assumed constant. The first panel reports results for two stock market illiquidity measures. S1 is negative one times the Pástor and Stambaugh (2003) liquidity proxy; S2 is the log of the Hasbrouck (2006) latent common factor measure. The second panel reports results for two bond illiquidity series. B1 and B2 are logs of average price impact coefficients and bid-ask spreads, respectively, for on-the-run Treasury issues, as computed by Fleming (2003). Standard errors computed from the outer product of the gradient are shown in parentheses.

significant for the price impact measure (B1, in the table) and the skewness term is no longer significant for the bid-ask spread (B2). The magnitudes of both estimates are still relatively large however.

The additional panels of the table add other controls, while continuing to include those in the top panel. Given the earlier discussion of the likely effect of including additional endogenous variables, it is perhaps surprising that there is little diminution in the effects when contemporaneous returns are included. Including contemporaneous changes in expected turnover also has no effect. In line with the basic motivation of the model, the (unreported) coefficients in the mean on expected turnover are insignificant or positive (that is, illiquidity increases with increases in expected turnover) for all four series.

Besides the checks in Table 7, the tests have also been repeated with a range of alternative definitions of the underlying variables. These are omitted for brevity, but include equally weighting stocks in the latent common factor index; measuring bond supply by the outstanding amount of on-the-run issues; measuring bond volume with interdealer broker trades, or all trades; adjusting bond illiquidity measures for secular changes in GovPX share of trading; estimating expected volume with polynomial trends or trailing moving averages; and using ten-year bond returns and volatility together with the Treasury bill numbers. For none of these variations was δ_1 insignificant, and the magnitudes of the other coefficients were similar to those shown in the table.

To summarize, the model's predictions find consistent support in two very distinct markets, using very different liquidity measures, with different frequencies and time periods. The primary prediction of a positive association between realized turnover and liquidity variance is borne out in all the tests. The bond market series are short, resulting in noisy estimates. Still, the sign and magnitude of the secondary effects (δ_2 and δ_3) are mostly in line with the theory's predictions as well. For the longer stock market series, all of the estimated

Table 7
Alternative first moment specifications

Mean controls		S1	S2	B1	B2
$\Delta \log \mathscr{I}_{t-1}$,	δ_1	1.583	1.986	2.084	2.225
$\Delta \log \widehat{\sigma}_t$		(0.313)	(0.164)	(0.768)	(0.554)
	δ_2	0.493	0.718	1.452	−0.171
		(0.071)	(0.039)	(1.208)	(0.778)
	δ_3	4.732	0.949	−0.034	0.583
		(0.714)	(0.103)	(0.368)	(0.519)
$\Delta \log \mathscr{I}_{t-1}$,	δ_1	1.522	2.054	2.430	2.474
$\Delta \log \widehat{\sigma}_t$		(0.346)	(0.216)	(0.831)	(0.625)
$\Delta \log \mathscr{I}_{t-2}$,	δ_2	0.465	0.712	0.445	0.932
$\Delta \log \widehat{\sigma}_{t-1}$		(0.074)	(0.041)	(1.327)	(1.109)
	δ_3	5.057	0.770	0.089	0.584
		(0.957)	(0.105)	(0.385)	(0.564)
$\Delta \log \mathscr{I}_{t-1}$,	δ_1	1.274	1.911	2.076	2.236
$\Delta \log \widehat{\sigma}_t$,		(0.379)	(0.185)	(0.754)	(0.565)
$\Delta \log P_t$	δ_2	0.489	0.711	1.512	−0.207
		(0.074)	(0.039)	(1.193)	(0.817)
	δ_3	4.826	0.950	0.106	0.714
		(0.741)	(0.101)	(0.399)	(0.519)
$\Delta \log \mathscr{I}_{t-1}$,	δ_1	1.591	1.993	2.153	2.214
$\Delta \log \widehat{\sigma}_t$		(0.317)	(0.168)	(0.745)	(0.552)
$\Delta \log \overline{\mathscr{T}}_t$	δ_2	0.486	0.716	0.749	−0.029
		(0.073)	(0.039)	(1.176)	(0.908)
	δ_3	4.722	0.954	0.011	0.539
		(0.715)	(0.103)	(0.393)	(0.545)

The variance estimations of Table 6 are repeated with different specifications of the conditional mean of percent changes in illiquidity. The first panel employs a constant, contemporaneous changes in log volatility, and one lag of the dependent variable. The next three panels include those terms as well as the following variables: the second panel includes an additional lag of volatility changes and illiquidity changes; the third panel includes contemporaneous returns; the last panel includes percentage changes in expected turnover.

coefficients in the variance specification are highly statistically significant, and the economic magnitude of the implied effects is substantial.

5. Conclusion

This paper is concerned with the joint dynamics of volume and liquidity. Intuition suggests that the two quantities ought to be tightly linked: it should be easier to trade in more active markets. However, surprisingly, there is little evidence that the two covary *at all* over time. Studies (cited in the introduction) across a variety of asset classes, frequencies, and market microstructures typically find no association, or even a negative one.

I have argued that this lack of association is consistent with a paradigm that views liquidity as an aggregate property of a (frictionless) equilibrium. The willingness of agents to accommodate the marginal trading needs of others does not change as the scale of the transaction demand changes. I illustrate this in a simple model with stochastically overlapping generations and constant absolute risk aversion. Liquidity is determined by the aggregate size of the population, whereas volume is determined by the gross flux of entering and exiting agents.

I distinguish the model by deducing new predictions about the conditional second moment of liquidity. Other models may also be able to explain why there is not a positive relationship between volume and liquidity. But the reason why the present model does so is that it asserts that volume is equally necessary for positive and negative changes in liquidity levels. Thus, the economic insight the paper proposes is encapsulated in the prediction that the scale of liquidity innovations—liquidity risk—should be positively related to volume. Intuitively, large changes in liquidity cannot occur without a lot of population flux, and a small amount of flux must imply a small change in liquidity.

This logic appears to be borne out in tests using data from the U.S. government bond and stock markets. The effect seems both reasonably robust and significant in magnitude.

The tests provide strong support for the view that aggregate risk-bearing capacity may play an important part in liquidity determination. This view is not inconsistent with other factors—asymmetric information, search costs, and the financial constraints of intermediaries—also playing important roles. However, models incorporating these features would seem to face a challenge in confronting the joint time-series properties of volume and liquidity.

Finally, the new findings in this paper concerning liquidity risk may have important implications in their own right. They suggest that higher levels of activity may not unambiguously indicate healthier markets accommodating greater risk transfer. In fact, such conditions may also be indicative of greater susceptibility to sudden changes in liquidity.

Appendix A. Proofs

This appendix collects proofs of the results in the text.

Proof of Proposition 2.2. Setting $X^{(1)} = 1$ for notational convenience, all exiting agents have holdings $1/N_{t_{n-1}}$ and there are D_n of them. All entering agents must end up with $1/N_{t_n}$ shares and there are A_n of them. Their respective trade demands can be offset up to the minimum of the two: $\min(\frac{A_n}{N_{t_n}}, \frac{D_n}{N_{t_{n-1}}})$. The balance, $\left|\frac{A_n}{N_{t_n}}, \frac{D_n}{N_{t_{n-1}}}\right|$, must be traded with the remaining population. Total volume is therefore

$$\min\left(\frac{A_n}{N_{t_n}}, \frac{D_n}{N_{t_{n-1}}}\right) + \left|\frac{A_n}{N_{t_n}}, \frac{D_n}{N_{t_{n-1}}}\right| = \max\left\{\frac{A_n}{N_{t_n}}, \frac{D_n}{N_{t_{n-1}}}\right\}. \qquad \square$$

Proof of Proposition 2.3. Each agent chooses her risky asset holdings, α_t, consumption, c_t, and riskless asset holdings, b_t, to maximize

$$-\mathrm{E}_t\left[\int_t^\tau e^{-\rho u} e^{-\gamma\, c_u}\,\mathrm{d}u\right],$$

where τ is her death time. Equivalently, conditional on her own survival, the agent's problem is to maximize

$$-\widetilde{\mathrm{E}}_t\left[\int_t^\infty e^{-\rho^\star u} e^{-\gamma\, c_u}\,\mathrm{d}u\right],$$

where $\rho^\star \equiv \rho + \mu_1$ and, under the conditional measure, the population process has decay $\tilde{\mu}(N) \equiv \mu(N) - 1 = (N-1)\mu_1$.

Let $W_t = \alpha_t P_t + b_t$ be the agent's time-t wealth and $J = J(W_t, \Theta_t, N_t; t)$ be her value function (the maximized objective). Optimal policies must be chosen to be self-financing, which implies that W_t evolves according to

$$\mathrm{d}W_t = \alpha_t\,\mathrm{d}P_t + \alpha_t\Theta_t\,\mathrm{d}t + rb_t\,\mathrm{d}t - c_t\,\mathrm{d}t.$$

Following the usual solution procedure, we conjecture that the risky asset obeys the law

$$\mathrm{d}P_t = \mu_P\,\mathrm{d}t + \sigma_P\,\mathrm{d}Z_t + \Delta g(N)\,\mathrm{d}N_t,$$

where μ_P and σ_P are constants and $g(N)$ is a function, all to be determined in equilibrium. Then a necessary condition of optimal policies is that

$$\max_{c,\alpha,b}\left\{J_t + \mu_\Theta J_\Theta + \frac{1}{2}\sigma_\Theta^2 J_{\Theta\Theta} + (rW - \alpha P + \alpha\Theta + \alpha\mu_P - c)J_W + \alpha\sigma_\Theta\sigma_P J_{W\Theta} \right.$$
$$\left. + \frac{1}{2}\alpha^2\sigma_P^2 J_{WW} + (\lambda(N)\Delta^+ J + \tilde{\mu}(N)\Delta^- J) + u(c)\right\} = 0. \tag{5}$$

Here, $N \geqslant 2$ and

$$\Delta^+ J \equiv J(W + \alpha\Delta^+ g(N), \Theta, N+1),$$

$$\Delta^- J \equiv J(W + \alpha\Delta^- g(N), \Theta, N-1),$$

and $\Delta^+ g(N) \equiv g(N+1) - g(N); \Delta^- g(N) \equiv g(N-1) - g(N)$.

Next, we conjecture the existence of functions $\lambda(N)$ and $J(W, \Theta, N; t)$ satisfying the auxiliary equation

$$\begin{aligned}&\lambda(N)(\Delta^+ g(N))J_W(W + \alpha\Delta^+ g(N), \Theta, N+1)\\&\quad + \mu(N)(\Delta^- g(N))J_W(W + \alpha\Delta^- g(N), \Theta, N-1) = 0,\end{aligned} \tag{6}$$

when α has been chosen optimally.

If that holds, then the condition for such an optimal α choice is simply

$$(\Theta + \mu_P - rP)J_W + \sigma_\Theta \sigma_P J_{W\Theta} + \alpha\sigma_P^2 J_{WW} = 0$$

because (6) stipulates that the jump terms in the first-order condition cancel. Hence, we must have

$$\alpha = -((\Theta + \mu_P - rP)J_W + \sigma_\Theta\sigma_P J_{W\Theta})/\sigma_P^2 J_{WW}.$$

In addition, the first order condition for consumption stipulates $u'(c) = \gamma e^{-\rho^\star t} e^{-\gamma c} = J_W$.

Finally, we conjecture a form of the value function:

$$J(W, \Theta, N; t) = -e^{-\rho^\star t - r\gamma W + h(N)},$$

where $h(N)$ is to be determined. With this form, optimal consumption is

$$c = rW - h(N)/\gamma - \log(r)/\gamma$$

and $J_w/J_W W = -1/r\gamma$. So optimal asset demand satisfies

$$\alpha = \frac{\Theta + \mu_P - rP}{r\gamma\sigma_P^2}. \tag{7}$$

Market clearing then requires $X^{(1)} = N_t \alpha$, which implies

$$P = \frac{\Theta}{r} + \frac{\mu_P}{r} - \frac{\gamma\sigma_P^2}{N} X^{(1)}, \tag{8}$$

which, in turn implies

$$\mu_P = \mu_\Theta / r, \tag{9}$$

$$\sigma_P = \sigma_\Theta / r, \tag{10}$$

$$g(N) = -X^{(1)}\gamma\sigma_\Theta^2/(Nr^2). \tag{11}$$

This verifies the conjectured form of the price process.

With these results, Eq. (5) reduces to the difference equation

$$\begin{aligned}0 &= -1 + \frac{\rho^\star}{r} + \log(r) + h(N) + \frac{1}{2}\frac{p}{N^2}\\&- \frac{\lambda(N)}{r}(e^{\{\frac{-p}{N^2+N} + h(N+1) - h(N)\}} - 1) - \frac{\tilde{\mu}(N)}{r}(e^{\{\frac{p}{N^2-N} + h(N-1) - h(N)\}} - 1),\end{aligned} \tag{12}$$

where $p \equiv r\gamma^2\sigma_P^2(X^{(1)})^2$. Also, the auxiliary equation, (6), reduces to

$$\log\left(\frac{N-1}{N+1}\right) + \log\left(\frac{\lambda(N)}{\tilde{\mu}(N)}\right) - p\left(\frac{1}{N^2+N} - \frac{1}{N^2-N}\right) + (h(N+1) - h(N-1)) = 0. \tag{13}$$

To complete the proof, we must find functions $h(N)$ and $\lambda(N)$ to satisfy these coupled difference equations. If $\lambda(N)$ is substituted out, we have a second-order difference equation for $h(N)$, which can be written as a bivariate first-order system.

To deduce appropriate boundary conditions, we note that, for $N = 1$, the jump-risk terms in the demand equation will only cancel if $\lambda(1) = 0$. Hence, the $N = 1$ state is absorbing (under the subjective measure), and the value function is that of a single-agent economy with nonstochastic population. It follows that

$h(1) = -\log(r) + 1 - (\rho^\star/r) - p/2$. Likewise, for large N, the price function implies that prices and wealth must be independent of the population size. As $N \to \infty$, then, the solution converges to that of a risk-neutral representative agent, which entails $h \to -\log(r) + 1 - (\rho^\star/r) \equiv h_\infty$. Given these conditions, the system is well posed and can readily be solved, e.g., by relaxation methods.

Having found a solution and thus fully specified J, we can verify the transversality condition

$$\lim_{\tau\to\infty} \widetilde{\mathrm{E}}_t\{J_\tau\} = 0$$

by noting that (5) implies that the drift of the process J_t is $-u(c_t) = -rJ_t$. Hence, the process $\hat{J}_t \equiv e^{rt}J_t$ is a martingale, and

$$e^{rt}J_t = \widetilde{\mathrm{E}}_t\{\hat{J}_\tau\} = e^{r\tau}\widetilde{\mathrm{E}}_t\{J_\tau\}.$$

So

$$\widetilde{\mathrm{E}}_t\{J_\tau\} = J_t e^{-r(\tau-t)} \to 0.$$

Finally, we analyze the convergence rate of the solution for the arrival intensity $\lambda(N)$ by first writing (13) as

$$\lambda(N) \sim \tilde{\mu}(N)\frac{N+1}{N-1}(1 - O(\Delta h) + O(N^{-4})).$$

Putting this into (12), we can write that equation as

$$\begin{aligned} h(N) - h_\infty &\sim O(N^{-2}) \\ &\quad + \frac{\mu_1}{r} N\left[\left\{\frac{-p}{N^2+N} + h(N+1) - h(N)\right\} + \left\{\frac{p}{N^2-N} + h(N-1) - h(N)\right\}\right] \\ &\quad + N(O(\Delta h) - O(N^{-4}))\left\{\frac{p}{N^2+N} + h(N+1) - h(N)\right\} \\ &\sim O(N^{-2}) + N[O(\Delta^2 h) - O(\Delta h)O(N^{-2}) - O([\Delta h]^2)], \end{aligned}$$

where the last line drops dominated terms. We now argue that neither of the three terms in brackets can dominate for large N. Indeed, if $h(N) \sim O(N^{-\delta})$ for any $\delta \in (0,2)$, then $O(N\Delta^2 h) \sim O(N^{-\delta-1})$, $O(\Delta h/N) \sim O(N^{-\delta-1})$, and $O(N[\Delta h]^2) \sim O(N^{-2\delta-1})$. This is a contradiction since these orders all differ from the assumed order of h. That leaves the possibility that some bracketed term converges slower than $O(N^{-\delta})$, $\forall\delta > 0$. But that condition implies that along some sequence of (pairs of) integers, $\delta \log N > \Delta \log|h|$ or $\delta/N > |\Delta h/h|$. This implies $N|\Delta h/h|$ can be made arbitrarily small, and hence $O(\Delta h) < O(h)/N$. A similar argument shows $O(\Delta^2 h) \leqslant O(h)/N$. These bounds again imply that the orders of $O(N\Delta^2 h)$, $O(\Delta h/N)$, and $O(N[\Delta h]^2)$ are strictly smaller than that of h.

We conclude $h(N) - h_\infty \sim O(N^{-2})$ so that $\Delta h \sim O(N^{-3})$. Returning to the lambda equation, this implies

$$\frac{\lambda(N)}{\tilde{\mu}(N)}\frac{N-1}{N+1} = \frac{\lambda(N)}{(N+1)\mu_1} \sim (1 - O(N^{-3}))$$

as asserted. □

Appendix B. Further properties of the model economy

The paper is concerned with the joint dynamics of illiquidity and volume. In assessing the underlying theory of liquidity, it is also of interest to evaluate other dynamic relationships it implies. This appendix presents evidence on the covariances between volume, liquidity, returns, and volatility. For each pair, the intuition captured in the model is briefly discussed and the association is illustrated via simulation.

In undertaking this exercise, it is important to bear in mind the inherent limitations of arithmetic CARA economies from which the model necessarily suffers. As discussed in the text, the economy is not stationary because both of the driving processes, dividends and population, are random walks. So no unconditional moments actually exist. This means that the magnitudes of the effects illustrated here are only formally meaningful conditional on the starting level of the processes and the time horizon.

Table 8
Simulation: parameters

	T	μ_1	N_0	r	γ	$X^{(1)}$	Θ_0	μ_Θ	σ_Θ	p	P_0
Stock case	1/12	0.40	10	0.01	0.094	7.111	1.150	0.00	0.150	1.0	1.0
Bond case	1/52	4.00	10	0.01	1.467	0.852	1.080	0.00	0.080	1.0	1.0

The table gives the parameters of the two simulation cases. The symbol definitions are as in Section 2.

Another problem for CARA economies is that prices can become negative because dividends are not bounded below. (Also, the risk premium could drive prices negative for low population levels.) It is common in the literature to handle this by never scaling by price, i.e., interpreting raw price changes as "returns" and their standard deviation as "volatility." Alternatively, one can compute percentage changes as usual, conditional on the path of the economy not taking prices to zero. I adopt the latter approach in the simulations here. This results in no survivorship bias because prices stay positive for the cases examined.

B.1. Parameters

For purposes of simulation, I fix parameter values within the range investigated in Section 3 and, to the extent possible, consistent with the sample moments reported in Table 5. Rather than undertake an exhaustive exploration, I examine two cases to illustrate the intuition. Of course, no claim can be made as to the generality of the numbers reported.

The two cases are parameterized to roughly correspond to the stock and bond data. The exact choices are shown in Table 8. From Table 1, the level of turnover is directly proportional to the departure intensity per agent, μ_1. The stock and bond cases use $\mu_1 = 0.4$ and 4, respectively, yielding annual turnover numbers of approximately 80% and 800%. (The logs of these are, respectively, -0.22 and 2.08, which can be compared to the values in Table 5.)

The initial population value, N_0, then determines the volatility of liquidity. It turns out that $N_0 = 10$ is approximately what is needed for both markets. High liquidity risk requires very volatile populations. And, conditional on the Poisson intensities just selected, the population size must be quite small. In effect, the model portrays a market characterized by a few impatient players.

The rest of the parameters are chosen to satisfy five conditions. First, I set $r = \rho = 0.01$ in line with values used in Section 3. Likewise, I maintain the assumption that $p \equiv r(\gamma\, X^{(1)}\, \sigma_P)^2 = 1$, which was taken for numerical convenience in solving for the arrival intensity function. Next, the initial level of dividends is chosen to have the initial price be $P_0 = 1.00$ when dividend growth is $\mu_\Theta = 0$. These latter choices are arbitrary normalizations that have no effect on the dynamics. One more degree of freedom is used to match the levels of illiquidity, e.g., to the average levels of (log) trading cost in the second and fourth panels of Fig. 4.

The last consideration in selecting parameters is the relative magnitude of the flow-driven component of price moves as compared to the dividend-driven component. The price of the risky asset is the sum of two terms: one proportional to the dividend level and having volatility σ_P/P and the other proportional to the inverse population whose volatility is the same as that of liquidity. The degree of "excess" volatility is determined by the relative sizes of these terms. I select parameters that add from 8% to 35% to dividend volatility. This level seems reasonably conservative while still yielding discernible effects.[23]

Table 9 shows some resulting summary statistics for simulated data generated from the parameterized models. For each case 100,000 paths are generated, with a time increment of 1/8 of a day. For purposes of computing realized volatility, returns are cumulated to daily frequency. Trading is assumed to take place continuously when computing turnover. Turnover, volatility, and illiquidity risk are reported as annualized percentages. The initial level of liquidity is reported in elasticity units, i.e., percentage cost per

[23]Brandt and Kavajecz (2004) report that flow regressions explain up to 26% of the variance of Treasury bond price changes, which suggests that the choices here are in the right range.

Table 9
Simulation: sample properties

	Turnover	Volatility	Initial illiq	Illiq risk	Excess vol	Survival pct
Stock case	78.4	14.5	33.0	28.9	1.080	100
Bond case	733.5	9.5	0.60	90.6	1.349	100

The table shows sample moments from 100,000 replications of each case. Turnover is the mean total trade per interval (in annualized percentage) assuming continuous trade. Volatility is the mean of the realized standard deviations of daily returns (in annualized percentage). Initial illiquidity is $\mathscr{I}$ in basis points at the start of the interval. Illiquidity risk it the standard deviation of percent changes in $\mathscr{I}$ (in annualized percentage). Excess volatility is the ratio of the volatility in the second column to the volatility obtained in an identical simulation with population fixed at N_0. Survival is the percent of sampled paths along which the population and prices stayed strictly positive.

Table 10
Simulation: correlations

	$\rho(\mathscr{I}, R)$	$\rho(\mathscr{I}, \widehat{\sigma_R})$	$\rho(\sqcup, R)$	$\rho(\mathscr{T}, \widehat{\sigma_R})$	$\rho(R, \widehat{\sigma_R})$
Stock case	−0.276	0.124	−0.0227	0.288	−0.171
Bond case	−0.679	0.196	−0.0463	0.440	−0.177

The table shows pairwise correlations between returns (R), percent changes in illiquidity ($\mathscr{I}$), turnover ($\mathscr{T}$), and realized volatility ($\widehat{\sigma_R}$) in 100,000 simulations using the parameters shown in Table 8.

percentage of supply traded. Excess volatility is the ratio of volatility to what would obtain if the population were fixed at N_0.

The turnover, volatility, and illiquidity risk numbers are roughly in line with the moments in the data, although note that the volatility in the bond case has been chosen to be closer to that of a long-dated Treasury bond, rather than a T-bill. The last column in the table verifies that all of the simulated price paths stayed positive.

B.2. Illiquidity correlations

Table 10 shows the sample correlation coefficients that result from the simulations. Consider first the dynamics of illiquidity.

The model has a built-in relationship between illiquidity and returns. Increases in illiquidity occur if and only if the population decreases, and this decrease raises the risk-discount term in asset prices. That accounts for the negative correlation shown by the simulations. An additional prediction of the model is that the association is stronger in down markets, which is due simply to the convexity of the risk term: a decrease in N of one has a bigger percentage effect on $1/N$ than an increase.[24] This effect is amplified, in terms of covariances, by the higher upside volatility of illiquidity and higher down-side volatility of returns (i.e., the skewness of each). Positive bond return paths exhibit a covariance of -1.84 between returns and illiquidity, whereas negative paths produce a covariance of -3.70 (the equivalent numbers for the stock case are -0.39 and -0.59, respectively).

Next, the relationship between illiquidity and volatility is positive in the model for reasons discussed in the text. The rate of order flow in the model is constant, but the degree to which that flow moves prices depends upon illiquidity. When illiquidity is low, price pressure does not affect the market. But when illiquidity is high, trades move prices, and volatility is higher as a result. This positive correlation is a prominent feature in all empirical studies of illiquidity.

B.3. Turnover correlations

Now consider the dynamics of turnover. The third column in Table 10 shows that the model does not imply an economically large correlation between turnover and returns. This is a shortcoming: in most markets,

[24] I am grateful to the referee for pointing out this feature and for highlighting several of the relationships examined in this appendix.

volume goes up when prices do. That effect could be incorporated by specifying intensity functions $\mu(N)$ and $\lambda(N)$ that rise faster than linearly in N. The model could still be solved with this alteration without affecting either the first- or second- moment predictions about volume and liquidity.

The fourth column of the table shows that the model delivers one other empirical regularity: a positive correlation between volatility and volume (or turnover). This is straightforward. High realized volume means there are higher than average gross flows (arrivals plus departures). Since flows move prices, this produces more realized volatility.

It is interesting to note that the model implies volume and volatility *only* covary when the volatility is due to flows, or, equivalently, to changes in risk premia. Dividend innovations, by contrast, produce no volume. This suggests an insight into the question of why some large price moves are accompanied by high volume but others are not.

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