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Forecast Dispersion and the Cross Section of Expected Returns

TIMOTHY C. JOHNSON*

ABSTRACT

Recent work by Diether, Malloy, and Scherbina (2002) has established a negative relationship between stock returns and the dispersion of analysts' earnings forecasts. I offer a simple explanation for this phenomenon based on the interpretation of dispersion as a proxy for unpriced information risk arising when asset values are unobservable. The relationship then follows from a general options-pricing result: For a levered firm, expected returns should always decrease with the level of idiosyncratic asset risk. This story is formalized with a straightforward model. Reasonable parameter values produce large effects, and the theory's main empirical prediction is supported in cross-sectional tests.

IN AN INTRIGUING RECENT ARTICLE, Diether, Malloy, and Scherbina (2002) (hereafter DMS) document a new anomaly in the cross section of returns: Firms with more uncertain earnings (as measured by the dispersion of analysts' forecasts) do worse. The finding is important in that it directly links asset returns with a quantitative measure of an economic primitive—information about fundamentals—but the sign of the relationship is apparently wrong. Rather than discounting uncertainty, investors appear to be paying a premium for it. This would seem to pose a formidable challenge to usual notions of efficiently functioning markets.

This article argues that the challenge can be met. In fact, a simple, standard asset pricing model implies the DMS effect even when there is no cross-sectional relationship between dispersion of beliefs and fundamental risk. The logic relies on two elements. First, when fundamentals are unobservable, dispersion may proxy for idiosyncratic parameter risk. Second, for a levered firm, expected equity returns will in general decrease with the level of idiosyncratic asset risk due to convexity. I formalize this in a straightforward way. The story has some direct and distinguishing testable implications, which I take to the data. The empirical evidence is remarkably supportive.

The theory offered here contrasts sharply with the explanation suggested by DMS. They view the negative relationship between forecast dispersion and subsequent returns as supportive of a story in which costly arbitrage leads

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to mispricing when agents have differing beliefs. With short-sales constraints, the argument goes, the most optimistic investors will bid prices up too high (assuming they do not adjust for the winner's curse). Hence the more views differ, the more stocks may become overpriced.[1]

This intuition may well be valid. Short-sales constraints, heterogeneous information, and investor biases are certainly important features of real markets that undoubtedly affect price formation. However, this paper demonstrates that they need not be necessary to explain the return puzzle.

My model does not invoke any market frictions or irrationality. This does not make its description incompatible with the DMS story: Both could be contributing to the effect. Instead, it illustrates an alternative economic mechanism that may be important in its own right.[2] Moreover, the frictionless approach makes it possible to develop a formal model whose parameters can be calibrated or estimated, and whose dynamic implications can be rigorously quantified. Most importantly, the model provides falsifiable predictions that are, in fact, verified.

Taken together, the model and the tests illustrate the power of classical building blocks (in this case, the theory of unobserved state variables and the contingent claims analysis of capital structure), when combined, to deliver rich implications about patterns in asset returns. The paper extends the line of research including Berk, Green, and Naik (1999), Berk and Green (2002), Johnson (2002), Carlson, Fisher, and Giammarino (2002), Gomes, Kogan, and Zhang (2002), and Brennan, Wang, and Xia (2002), which seeks ways to account for the main features of the cross section without invoking systematic mispricing.

The approach here is not without its limitations, however. First, the paper addresses one anomaly in isolation. Since a simple one-state-variable model suffices for my purposes, I avoid embellishments that might increase the scope for capturing additional dimensions of the data. Second, I do not explicitly model information production/acquisition or capital structure choice, which are both clearly endogenous aspects of the problem. Although my empirical work will try to take that endogeneity into account, the model is only a partial-equilibrium account of return determination. Finally, in the same way, the theory is partial-equilibrium in taking the fundamental discount factor process of the economy as given, and the tests offer no new ways of identifying that process. Although the paper is able to show the linkage between the return patterns and risk exposures, it makes little or no contribution to our understanding of the nature of that risk.

The outline of the paper is as follows. Section I presents a brief discussion of the background literature and the connection between forecast dispersion, risk, and uncertainty. Section II introduces the asset pricing model, derives the main dynamic quantities, and provides some numerical illustrations. The primary testable implications are then derived, and these are tested

[1] For this to translate into a statement about *returns*, the overpricing needs to be corrected. So the story actually implies that *changes* in dispersion should be correlated with returns.

[2] That said, the two explanations can be empirically distinguished. As DMS fully acknowledge, the evidence tying the dispersion effect in returns to short-sales costs is mixed at best.

in Section III. Section IV summarizes the contribution and considers some further implications.

I. Forecast Dispersion

Since 1983, IBES Corporation has been recording the earnings forecasts of Wall Street analysts for a large cross section of U.S. stocks. Each observation in this data set is the issuance of a new forecast (a point estimate) by an individual analyst for a particular stock's operating earnings for a particular reporting period (e.g., the "current" fiscal year). At any given date, one can readily compute the cross-analyst standard deviation of all outstanding forecasts pertaining to a single stock and horizon. Comparing this measure of disagreement across stocks (after suitably normalizing for scale differences), one finds substantial variation. This section considers what is really being measured and how the difference across firms should be interpreted.

The first point to note is that what is *not* being measured is a subjective assessment of earnings uncertainty. That is, we have no data on how confident any of the forecasters is about his/her point estimate. In theory, a group of forecasters could all agree about the mean of the distribution while each being hugely uncertain. Conversely, they could all place enormous confidence in their own estimates while differing wildly from each other.

Despite this possibility, it is common practice throughout the social sciences to view the variation across a survey of respondents' assessments of an unknown quantity as a proxy for the true uncertainty in their environment. The intuitive basis of this interpretation is clear enough: An observer of the survey with no other information about the quantity (or about the respondents) might well view each estimate as being as good as the next, and so treat them as separate noisy signals of the true value. In aggregating this information, this observer would then construct a posterior distribution whose variance would be directly proportional to the variance across signals.

While it seems quite natural, this multiple-signal interpretation raises a number of questions. Do the reported assessments in fact represent all relevant sources of information available to agents? If so, why do the forecasters not themselves aggregate their joint information? Are they, in fact, reporting unbiased views subject to random perturbations, or are the estimates, and indeed, the level of dispersion itself a function of their incentives?

The latter issue may be especially important in dealing with IBES data. Not only are analysts' forecasts known to be biased by conflicts of interest, but their choice of whether to conform to or deviate from consensus might well be influenced by career concerns. This realization highlights the need for a more complete understanding of exactly how forecast differences arise. On the other hand, since the focus of this paper is on *differences* in dispersion across stocks, issues of bias may matter somewhat less.

Moreover, there is, in fact, empirical support for viewing disagreement as a proxy for uncertainty. Measures of respondents' uncertainty are explicitly elicited by the NBER-ASA survey of economic forecasters. This permits a direct comparison with dispersion of the same agents' forecast means. Zarnowitz and Lambros (1987) find a significant positive association between the two for forecasts of GNP and the GNP deflator. Bomberger (1996) compares the dispersion of inflation forecasts from the Livingston survey to subsequent realized variance of forecast errors and also finds a strong positive relationship. These studies focus on the time-series interaction between uncertainty and dispersion, and their subjects are presumably disinterested. So caution is warranted in extrapolating to the current setting. Still, the findings do suggest that the intuition linking the two concepts is on solid ground. This paper will build on that foundation, taking that linkage as a maintained hypothesis.[3]

In the next section, I will formalize the multiple-signal model. When dispersion arises from separate sources of information about an unobservable fundamental process, it becomes natural to distinguish between two components of the total uncertainty facing investors. The stochastic evolution of the underlying value itself is primitive to the economy in that it is independent of the informational environment. This variability might be called fundamental risk. In contrast, the uncertainty about the current value of that process is purely determined by the information setting. I call this parameter risk. Forecast dispersion, under the above interpretation, proxies only for this second component.

Why should firms differ in their degree of parameter risk? At least two distinct factors are involved. First, some businesses are inherently harder to assess than others. Second, firms, being themselves the source of most of the relevant information, can choose how much of it to provide. In both respects, parameter risk clearly goes well beyond uncertainty about current accounting earnings. But even on this one dimension, firms range from predictable, simple, and transparent to unfamiliar, complex, and opaque.

While substantial cross-sectional variation in parameter risk is thus to be expected, from the point of view of financial theory, there is no obvious reason why agents should care about it. Almost by definition, the noisiness of signals has no direct connection to the riskiness of a firm's cash flow. Nor would it seem likely to have a systematic, nondiversifiable character. This is an empirical question, and here the results of Diether, Malloy, and Scherbina (2002) are directly relevant. In fact, their findings would appear to suggest that parameter risk is "priced"—but that investors prefer to bear *more* of it, not less. The model developed below will show that this may be misleading, however. I will not assume any hedging role for parameter risk, and will regard it as idiosyncratic, in a sense to be made precise.

To summarize, the paper's stance is that analyst forecast disagreement is likely to be a manifestation of nonsystematic risk relating to the unobservability of a firm's underlying value. Having distinguished between fundamental and parameter risk, and between priced and unpriced risk, I should draw one further distinction between the exogenous risks facing investors and the risk

[3] It should be pointed out that the behavioral story advanced by DMS does not rely on this interpretation, and instead views dispersion more literally as a measure of heterogeneity of beliefs.

characteristics that emerge endogenously in the prices of traded claims. It is not yet clear, for example, whether one should expect firms with high forecast dispersion to therefore have high stock price volatility. Determining that relationship and deducing the connection between dispersion and expected returns requires a full valuation model, which is the subject of the next section.

II. The Model

The model I will use embeds a simple valuation framework in an environment of partial information. I first describe that environment, and then deduce the dynamics of asset prices. Finally I present some numerical illustrations that verify that reasonable parameter values produce reasonable effects.

A. Information

Consider a single firm whose true value is an unobservable diffusion process. (The meaning of true value will be defined below.) There are N distinct signals of this process available to investors, which intuitively correspond to the output of separate research analysts. Formally, let the value process be V_t and the n^{th} signal be $U_t^{(n)}$. Then the specification I assume is

$$dV_t/V_t = \epsilon\, dt + \sigma_V\, dW_t^V,$$

$$U_t^{(n)} = V_t e^{\eta_t^{(n)}},$$

where ϵ, the expected earnings rate per unit V, is known, and $\eta_t^{(n)}$, the n^{th} noise process, is unobservable. Let $\bar{\epsilon} \equiv (\epsilon - (1/2)\sigma_V^2)$. Then, in logs,

$$dv_t = \bar{\epsilon}\, dt + \sigma_V\, dW_t^V, \tag{1}$$

$$du_t^{(n)} = dv_t + d\eta_t^{(n)}, \tag{2}$$

$$d\eta_t^{(n)} = -\kappa\eta_t^{(n)}\, dt + \sigma_\eta\, dW_t^{\eta^{(n)}}. \tag{3}$$

Here investors observe the (log) level of fundamentals corrupted by stationary noise processes, or equivalently, they see N cointegrated signals of the true level that they care about.[4] Since V_t and $U_t^{(n)}$ are levels, dv_t can be thought of as the

[4] Filtering problems are often written in terms of unobserved *growth rates* of an observable process (rather than a level). The current problem can be equivalently expressed in that form as well. If we define the drift of u_t (supressing the superscript) as μ_t, then

$$\mu_t = \bar{\epsilon} - \kappa(u_t - v_t).$$

Hence the system can be written

$$d\mu_t = \kappa(\bar{\epsilon} - \mu_t)\, dt + \sigma_\eta\, dW_t^\eta,$$

$$du_t = \mu_t\, dt + \sigma_U\, dW_t^U,$$

with $\sigma_U\, dW^U \equiv \sigma_V\, dW^V + \sigma_\eta\, dW^\eta$. Inferences about v_t can then be made indirectly from inferences about μ_t because $v_t = u_t + (\mu_t - \bar{\epsilon})/\kappa$.

instantaneous earnings over dt and $du_t^{(n)}$ as the n^{th} analyst's time-t estimate of this earnings rate. (While I sometimes refer to these estimates as "forecasts," in fact, analyst estimates for the "current" fiscal period do pertain to a contemporaneous time interval.) How exactly analysts arrive at these numbers is not modeled here.

Agents, of course, will aggregate the information in all the signals in making inferences about V. For present purposes there is not much insight to be gained by keeping track of N of them. But the point to note is that when there are multiple analysts, the parameter σ_η will control the dispersion observed across their forecasts. (That dispersion will actually equal σ_η in the special case that the N processes $W_t^{\eta^{(n)}}$ are independent.) In thinking about a cross section of firms then, the interpretation I have in mind is that this is the dimension along which they differ. More realistically, the correlation structure across signals and the mean reversion parameter κ would also be expected to differ. And these too contribute to the total parameter risk (as will be shown below). So, even within this model, forecast dispersion is not a perfect proxy for information quality. But, having made the connection explicit, I will now drop the superscript and analyze the case of a single scalar signal U_t that can be thought of, without loss of generality, as the aggregate information available to investors.

Investors will also be assumed to know the variances and covariances of all the processes. Most importantly, they know the true fundamental volatility σ_V, even though the observation process U_t may be more or less volatile than V_t. In real life, perhaps, companies might try to smooth earnings by arranging for a negative correlation between W_t^η and W_t^V (i.e., issuing positive disinformation dW^η when the true shock dW^V is negative and vice versa). But this will not fool investors about true risks, and hence this correlation ends up having no interesting effects. Where it simplifies the exposition, I will set it to zero, which is thus also essentially without loss of generality.

In addition to the signal, investors can also observe the aggregate state of the economy as summarized by the stochastic discount factor process Λ_t that obeys

$$d\,\Lambda_t/\Lambda_t = -r\,dt + \sigma_\Lambda\,dW_t^\Lambda,$$

where r is the (known, constant) risk-free rate. This process conveys some information about the unobservable earnings because earnings have a systematic component: dW^V is correlated with dW^Λ.

It also streamlines some of the formulas if the signal noise dW^η is not correlated with dW^Λ, meaning that η is entirely unsystematic. And, although I stated above that it would be important for the argument that parameter risk be unpriced, it turns out in the current model *not* to be necessary to restrict the Λ–η correlation to achieve the desired result. I will return to this below. Again, the intuition is just that, when investors know the true riskiness of the true value, they do not get confused if there is some systematic component of the distortion in their news sources.

That completes the description of the informational setting. Investors have two Gaussian sources of information (the processes u_t and $\log \Lambda_t$) about a third Gaussian process. Standard filtering results (e.g., Lipster and Shiryaev (1977), Theorem 12.7) can now be applied to deduce their posterior beliefs about v_t given the history of these signals and given their prior beliefs. If their prior distribution is Gaussian, then their posterior distribution will always be Gaussian as well, and so can be summarized by the posterior mean and variance, denoted $\tilde{m}_t$ and $\tilde{\omega}_t$. The latter evolves deterministically and converges to a steady-state value $\tilde{\omega}$. The former is a stochastic process that evolves according to

$$d\tilde{m}_t = \bar{\epsilon}\, dt + \tilde{h}_t d\widetilde{W}_t, \tag{4}$$

where $\widetilde{W}_t$ is a scalar Brownian motion process summarizing the information in U and Λ about V, and $\tilde{h}_t$ is another deterministic process that converges to σ_V in the steady state. (Explicit expressions for $\tilde{\omega}_t, \tilde{h}_t$, and $\widetilde{W}_t$ are given in the Appendix.) Since the theory developed here has nothing much to say about prior information, I will assume from now on that enough time has elapsed for $\tilde{\omega}_t$ and $\tilde{h}_t$ to have converged to their long-run values. In that case, investors' beliefs are summarized by the single state variable $\tilde{m}_t$, the conditional expectation of $v_t = \log V_t$.

Below I will also use the fact that time-t beliefs about future values v_T for any $T \geq t$ are also Gaussian. Indeed, since $\tilde{E}_t[v_T] = \tilde{E}_t[\tilde{E}_T[v_T]] = \tilde{E}_t[\tilde{m}_T]$ and similarly $\tilde{E}_t[v_T^2] == \tilde{E}_t[\tilde{\omega} + m_T^2]$ (where $\tilde{E}_t$ denotes expectation with respect to investors' time-t information), conditional beliefs at t about v_T are distributed as

$$\mathcal{N}\big(\tilde{m}_t + \bar{\epsilon}\tau, \tilde{\omega} + \sigma_V^2 \tau\big), \tag{5}$$

where $\tau \equiv (T - t)$. This follows from (4) and the assumption that $\tilde{h}_t$ has converged to σ_V.

The only other quantities that will be needed are the correlations under the investors' information set between the exogenous processes dW_t^U and dW_t^Λ and the innovations process $d\widetilde{W}_t$. If dW_t^V has correlation $\rho_{V\Lambda}$ with dW_t^Λ and correlation ρ_{VU} with dW_t^U,[5] then the important result is that $d\widetilde{W}_t$ inherits these same correlations. In particular, this means $d\tilde{m}_t$ has the same amount of systematic risk, $\rho_{V\Lambda}\sigma_V$, as does the fundamental value process dV_t. This fact does not require that the noise $d\eta_t$ be orthogonal to either dV_t or $d\Lambda_t$.

B. Asset Prices and Expected Returns

The process V_t described above as "fundamental value" is neither fundamental nor valuable until it is given economic content by connecting it to some cash flows. The most straightforward way to do this is to imagine that V_T is paid

[5] The scalar Brownian motion dW_t^U (which I did not define above) is implicitly defined (along with σ_U) by $\sigma_U\, dW_t^U = \sigma_\eta\, dW_t^\eta + \sigma_V\, dW_t^V$. This is where it becomes convenient to take η orthogonal to V. Then $\sigma_U^2 = \sigma_\eta^2 + \sigma_V^2$ and $\rho_{VU} = \sigma_V/\sigma_U$.

out to the owners of V at some future date T (via a liquidation or takeover, for example). This means introducing a somewhat artificial new parameter, but has the virtue of yielding closed-form solutions for all the quantities of interest.

Moreover, with a finite horizon it becomes just as straightforward to value levered claims to the payoff V_T and risky debt secured by V_T along the lines of the classic Merton (1974) model. It turns out this is not merely a gratuitous generalization. The presence of debt has crucial implications for the relationship between parameter risk and expected returns.

To start, though, consider the price S_t of an unlevered claim to V_T. By the definition of the stochastic discount factor, this is

$$\tilde{E}_t[V_T \Lambda_T]/\Lambda_t.$$

Given investors' time-t information, V_T is conditionally log-normal according to equation (5) above. Since Λ_T is also log-normal, the above expectation evaluates to

$$S_t = e^{-r\tau} e^{\tilde{m}_t + (1/2)\tilde{\omega}} e^{(\epsilon + \rho_{V\Lambda}\sigma_V\sigma_\Lambda)\tau}. \tag{6}$$

In a recent article, Pástor and Veronesi (2003) use a similar model of parameter risk to derive an expression analogous to (6). As they point out, the presence of parameter risk serves to *raise* stock prices, as can be seen here from the presence of the $\tilde{\omega}$ term. However, this is entirely a static effect. It has no impact at all on expected returns, which can be seen by applying Itô's lemma to (6) and using the characterization of $d\tilde{m}_t$ in (4) to get

$$dS_t/S_t = (r + \pi)\,dt + \sigma_V\, d\widetilde{W}_t, \tag{7}$$

where $\pi \equiv -\rho_{V\Lambda}\sigma_V\,\sigma_\Lambda$ is the risk premium.

In this model, since investors' information about v is summarized by its conditional expectation $\tilde{m}$ that inherits v's covariances, the risk premium and volatility of S are just those that stem from fundamentals. Parameter risk affects neither.

Interestingly, this conclusion is not driven by the assumption that the noise process η is idiosyncratic. Both (6) and (7) are correct in the general case. Instead, what is important is how the parameter risk generated by η is *resolved*. Here it is resolved all at once when V_T is paid out (and hence revealed). This will result in S_T jumping to V_T at the final instant (and note that (7) is valid only for $t < T$ prior to the jump). But the key observation is that the jump is necessarily unpriced because of the assumption that the stochastic discount factor is a diffusion.

Notice that here the time T is actually playing two distinct roles: It is the time when both cash flows are realized and parameter uncertainty is resolved. A natural generalization would envision separation of these roles, with periodic reporting (perhaps noisy) of V at some sequence of dates, and cash flows (dividends) at another sequence. In this case, the stock price would follow a jump-diffusion and the degree of parameter risk would certainly contribute to the time-series variability of returns. There would still be no expected return effect, however, unless marginal utility could also jump at the reporting dates.[6]

Now consider a levered firm. Let K be the face amount of zero-coupon debt secured by V_T payable at T. Evaluating the price of the residual equity claim, P_t, by direct integration yields the familiar form

$$P_t = S_t\Phi(d_1) - e^{-r\tau}K\Phi(d_1), \tag{8}$$

where $\Phi(\cdot)$ is the cumulative normal distribution function,

$$d_1 = \frac{\log(S_t/e^{-r\tau}K) + \tilde{\sigma}^2/2}{\tilde{\sigma}},$$

$$d_2 = \frac{\log(S_t/e^{-r\tau}K) - \tilde{\sigma}^2/2}{\tilde{\sigma}},$$

and

$$\tilde{\sigma}^2 = \tilde{\omega} + \sigma_V^2\tau.$$

This is of course exactly the Merton (1974) model, except for one thing. The variance that matters, $\tilde{\sigma}^2$, is not just the cumulative time-series variability of V (or S) but includes the parameter risk as well.[7]

Again applying Itô's lemma, the dynamics of P_t are

$$dP_t/P_t = (r + \pi\delta S_t/P_t)\,dt + (\sigma_V\delta S_t/P_t)\,d\tilde{W}_t, \tag{9}$$

where $\delta \equiv \Phi'(d_1) = \partial P/\partial S$ as usual.

When the firm has debt, its expected excess return changes in an important way. The risk premium π of the equivalent unlevered firm[8] is amplified by a gearing factor $\delta S_t/P_t$ reflecting the effective exposure to V (or S) per dollar of P. While this is a well-known result from elementary options pricing, perhaps less well known is the fact that this multiplier is *decreasing* in volatility.[9] This has the startling implication that *raising* the uncertainty about the underlying asset value of a levered firm while holding the asset risk premium constant—that is, adding idiosyncratic risk—*lowers* its expected returns. More unpriced risk raises the option value of the claim, which lowers its exposure to priced risk.

This observation is the heart of the paper. And it shows that the central hypothesis that explains the dispersion anomaly in returns is that dispersion is a measure of idiosyncratic risk. The information story I proposed is one way of delineating why it might be such a measure. But any other way of achieving

[6] This reasoning suggests that the strategy of attempting to bury bad news by reporting it on busy days is completely misguided. Investors will *only* want compensation for the information uncertainty if bad news might come on bad (aggregate) days.

[7] These results generalize in a straightforward way to the interesting case where both assets (V) and liabilities (K) evolve stochastically. The form of (8) will be the same, in analogy to the formula for an option to exchange one asset for another.

[8] Note that the unlevered claim S_t does not actually have to be a traded asset.

[9] The proof of this is simply a matter of calculus. So I will omit it.

that linkage will also account for the findings of Diether, Malloy, and Scherbina (2002).

Here unobservability of fundamentals (quantified by $\tilde{\omega}$) raises uncertainty about the final payoff without raising the risk premium. Notice, again, that what is required to make this work is that the *resolution* of the uncertainty (via the terminal cash flow) is nonsystematic. How the uncertainty arises does not matter. This is why η can be arbitrarily correlated with Λ without affecting the results. In the previous section, I indicated that having idiosyncratic parameter risk was in some sense essential to the argument. I can now be more precise: It is the parameter risk component of cash-flows that must be idiosyncratic to deliver the expected return effect.

The result that the risk premium amplification term declines with $\tilde{\omega}$ will be illustrated below, and its sensitivity to parameter values explored. But before turning to the computations, there is one crucial feature of this result that is already apparent: The effect vanishes if the firm has no debt. When $K = 0$ then $P = S$, and from (7), $\tilde{\omega}$ has no effect on expected returns. More generally, as K increases (for fixed $\tilde{m}$ or S), P becomes more option-like and its sensitivity to uncertainty rises, which means the dampening effect of dispersion on priced risk will be stronger. (This will also be illustrated below.) Here, then, is the major empirical implication of the theory. If the story is right, the effect reported by DMS should increase with leverage. Other things being equal, firms with less debt should exhibit less sensitivity of expected returns to forecast dispersion. Testing this assertion is the subject of Section III. Notice, for now, that there is nothing analogous to this effect in the behavioral short-sales-constraint story. So this prediction sharply distinguishes the two explanations.

C. Magnitude of the Effects

To explore the magnitude of the expected return effects in the model, I first compute the risk premium term $\delta S_t/P_t$ in (9) for various values of the firm-level parameters. Throughout I fix the risk-free rate at $r = 0.04$ and the volatility of the pricing kernel at $\sigma_\Lambda = 0.5$.

In Figure 1 the value of the unlevered claim to V is fixed[10] at $S = 100$ and the volatility of fundamentals is $\sigma_V = 0.2$. I choose the correlation $\rho_{V\Lambda}$ to imply a risk premium for the unlevered firm of 0.05. The four panels of the graph show the levered risk premium for different values of the time horizon T as the level of debt K and the amount of parameter uncertainty $\tilde{\omega}^{1/2}$ vary.

With any of these horizons, the model effects are large. For fixed values of $\tilde{\omega}$, expected excess returns rise steeply with leverage, as one would expect. The unexpected relationship is along the other axis. For a fixed debt level of 60 (say), an increase in uncertainty about $\log V$ from 20% to 70% lowers expected returns by between 2% and 8%, depending on T. DMS report a return differential of about 80 basis points a month between the first and fifth quintile portfolios

[10] Fixing S while varying other parameters is equivalent to varying the conditional expectation $\tilde{m}$ in an appropriate fashion.

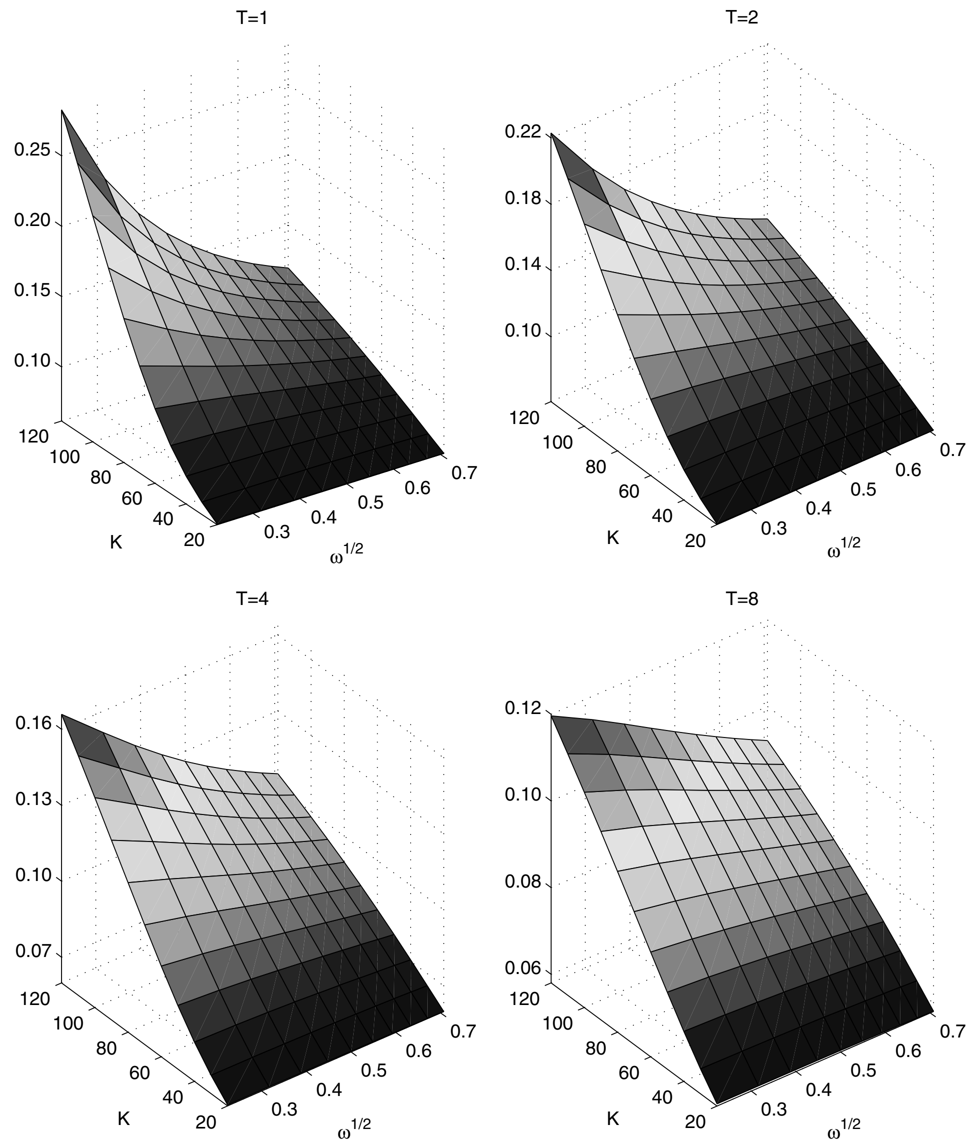

Figure 1. Risk premium for the levered firm. The figures show the expected excess return for a levered firm under the model of Section II. The different panels correspond to different choices of the cash-flow horizon T. The firm has a value of $S = 100$ and an unlevered expected excess return of 0.05. Face value of debt K varies along the left axis and the amount of parameter uncertainty $\tilde{\omega}^{1/2}$ varies along the right axis.

formed according to dispersion ranking. The theory here can easily generate differences of this magnitude. That a fully rational model with unexceptional parameter values can produce a strongly negative relationship between risk (of a particular type) and reward is remarkable.

The plots also vividly illustrate the main testable implication mentioned above: The effect of parameter uncertainty declines with debt. For an unlevered firm there is no $\tilde{\omega}$ sensitivity at all, implying that dispersion of beliefs should not affect expected returns. For firms closer to nominal insolvency, on the other hand, the effect can be extreme. The empirical work in the next section will thus focus on the sign of the cross second derivative.

One question the figure cannot address is what the range of values of parameter uncertainty is likely to be in practice. This matters because if reasonable information parameters cannot produce much variation in $\tilde{\omega}$, then it is unlikely that the model's effects are responsible for the variation in expected returns in the data. Likewise, it might be impossible to detect the hypothesized leverage interaction, unless in reality, firms do exhibit wide enough variation in parameter uncertainty.

Under the model, the steady-state posterior standard deviation $\tilde{\omega}^{1/2}$ is determined by three factors. First, the informativeness of the joint observation $(dU, d\Lambda)$ is governed by their correlations with dV. Second, these signals are redundant, and so are less informative together, to the extent that they are correlated with each other. Third, the level of the signal U is informative about V because the two are cointegrated (unlike V and Λ). The parameter governing the strength of this relation is κ, which can be interpreted as the decay rate of the error innovations $d\eta$. Equivalently, the half-life of these shocks is $\log 2/\kappa$. So if κ is large, U never wanders very far from V and inferences about fundamental value are easier to make.

To make things simple, I now will assume that η is uncorrelated with Λ and V. As discussed in Section II.A above, there is essentially no loss in generality in doing this, and it boils the covariance structure down to a single parameter $\rho_{VU} = \sigma_V/\sigma_U$ or equivalently $\rho_{\eta U} = \sigma_\eta/\sigma_U = \sqrt{(1 - \rho_{VU}^2)}$. Fixing the fundamental volatility σ_V at 0.2 as before, these correlations are then determined by the error volatility σ_η, which is a more intuitive quantity. Figure 2 plots the long-run level of uncertainty $\tilde{\omega}^{1/2}$ as a function of σ_η and the error half-life $\log 2/\kappa$. Recall that σ_η is the variable that would directly correspond to dispersion of analyst estimates under the multiple-signal interpretation. In the IBES data, it is not at all unusual to see analyst disagreements of over 50% or to see complete consensus (dispersion equal to zero). So the range of σ_η plotted seems realistic. For the other axis, one could perhaps interpret this half-life as the average length of time that analysts (and others) can be persistently wrong about a company's underlying financial picture. For the most transparent and forthcoming firms, this might be only a few months (e.g., between quarterly reports). At the other end of the spectrum, it is not at all hard (these days) to find examples of companies that have successfully misled investors for years.

As the figure shows, these ranges of parameters imply a long-run level of uncertainty about fundamental value of between 0% and 70% or so, which confirms that reasonable assumptions about information can produce a broad range along this dimension. In this context, I am reminded of Fischer Black's (1986) dictum that the stock market in general is probably "efficient" to within

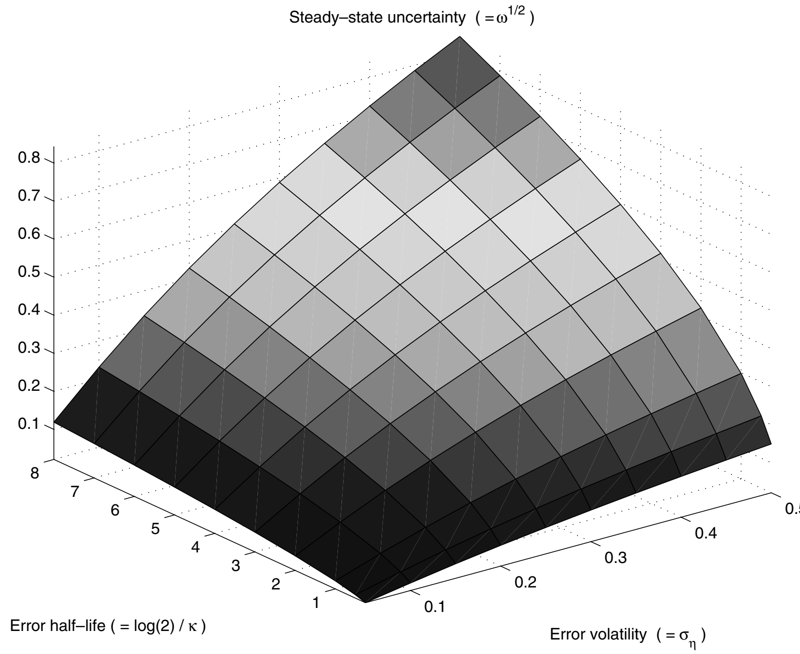

Figure 2. Determinants of parameter uncertainty. The figure shows the steady-state level of posterior standard deviation of beliefs, $\tilde{\omega}^{1/2}$, about log fundamental value as a function of the size and persistence of shocks to the noise process η_t. The size of shocks is measured by σ_η and their persistence by the half-life log $2/\kappa$. The plot assumes that η is uncorrelated with both the stochastic discount factor and the fundamental value process. See the appendix for the full expression.

a factor of 2. Interpreting this as a statement about the standard deviation of beliefs about fundamental values, a factor of 2 corresponds on a log scale to $\tilde{\omega}^{1/2} \approx 0.7$, which again suggests that the model is implying plausible levels of uncertainty.

This section has established that when firms have risky debt in their capital structure, their stock returns should be expected to decline as uncertainty about fundamentals increases. Furthermore, when that uncertainty is generated by a noisy-signal model with reasonable parameter values, the magnitude of the effects is easily large (and variable) enough to account for the patterns in the data. I now ask whether the theory can go beyond the anomalous patterns it was designed to explain and survive the test of its own new predictions.

III. Empirical Evidence

Suppose dispersion of analysts' beliefs is not systematically related across stocks to the fundamental systematic risk of underlying assets. In terms of the model's notation, this means that $\rho_{V\Lambda}^{(i)}\sigma_V^{(i)}$ does not covary with $\sigma_\eta^{(i)}$ or $\tilde{\omega}^{(i)}$ as i ranges across firms. In that case, the theory developed in the previous section for a single firm also implies that variation in dispersion across firms will, holding other things equal, also be associated with variation in expected returns because of the presence of debt in most firms' capital structure. For this to be a valid explanation of the findings of Diether, Malloy, and Scherbina (2002), it must be that the strength of this association increases in leverage. I now test this proposition empirically.

I follow the cross-sectional literature by implementing this test using standard Fama and MacBeth (1973) regressions. The main hypothesis then boils down to the assertion that the coefficient on an interaction term (i.e., the product) between a dispersion proxy and a leverage proxy ought to be significant and negative. Taken literally, the model offers an explicit nonlinear expression for expected returns for each firm. But the simplest specification captures the essence of the model's insight without leaning too heavily on the overstylized framework. Indeed, a glance at Figure 1 suggests that a formulation along the lines of

$$a + b \cdot \text{leverage} \cdot (c - \text{dispersion})$$

(with b, and $c > 0$) should well approximate the full model's expected return function. This approximation also points to a second prediction of the model: The dispersion effect should not enter on its own in the presence of the interaction term. In terms of economic intuition, this just captures the restriction that there should be no role for parameter risk when firms have no debt.

I carry out the test using the intersection of the CRSP, COMPUSTAT, and IBES universes for every month from January 1983 to December 2001.[11] The main econometric challenges are constructing meaningful proxies and holding other things equal.

My primary proxy for leverage will be book value of debt over the sum of book value of debt and market value of equity, which I call L. Later I will also construct a measure closer to the quantity that actually enters my model: face value of debt over total firm market value. This involves a model-based adjustment to book debt since debt market values are unavailable. Neither measure captures off-balance-sheet contributions to leverage, such as pension liabilities and derivative obligations.

My proxy for dispersion of beliefs is essentially the same as that used by DMS: the month-end standard deviation of current-fiscal-year earnings estimates across analysts tracked by IBES. The number and quality of analysts varies across firms. But DMS show that the expected return effects are not sensitive to the inclusion of controls for this heterogeneity. Those authors do document an important truncation bias in the standard deviation as computed by IBES. So accurate numbers must be recomputed from the detailed history of individual

[11] Overall these tend to be relatively large firms with above average past performance. A detailed discussion of the sample properties (and summary statistics) can be found in Diether, Malloy, and Scherbina (2002).

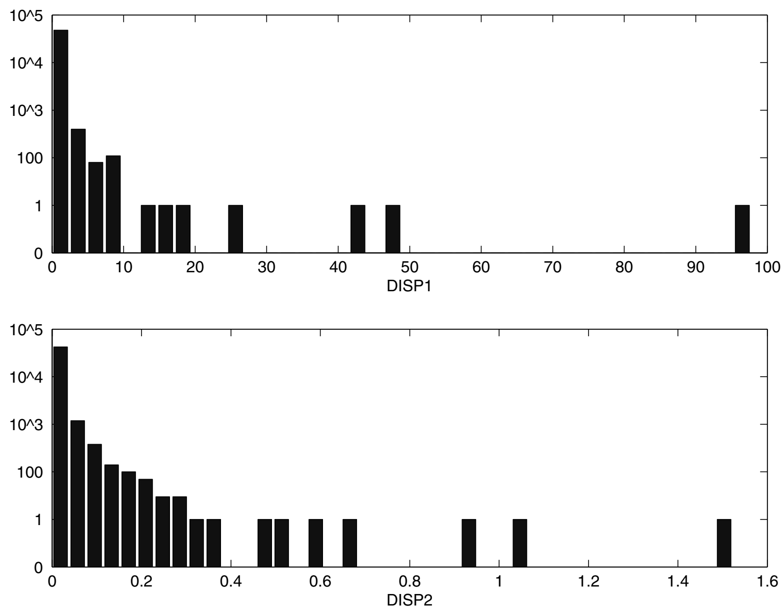

Figure 3. Raw dispersion measures. The figures show histograms of the normalized dispersion measures DISP1 and DISP2 for December 2000. The measure DISP1 is the standard deviation of analyst earnings estimates for the current fiscal year divided by the mean of the same estimates. The measure DISP2 normalizes instead by the most recently reported book value of assets.

forecasts. I use only the latest forecast made by each analyst, and I eliminate forecasts that are over 6 months old or that pertain to fiscal periods that have already ended.[12] DMS normalize each stock's standard deviation by the mean of the estimates, throwing out firm-months where this denominator is zero. I call this measure DISP1. Since this sacrifices some valid data and artificially inflates observations near zero, I also compute a measure, DISP2, that instead normalizes by book value of assets. This scaling more accurately represents the quantity dV/V that the model describes.

Figure 3 shows a histogram of these two measures for a typical month. Both measures display extreme right skewness. (Notice the log scale on the vertical axis.) This is problematic for two reasons. First, there is the general danger that outliers will distort OLS estimates. And second, for the particular specification at hand, it will give the regressions very low power to detect interaction effects. This is because the outliers all lie in the direction (high dispersion) for which the model expected return surfaces flatten out and the cross-derivative

[12] I make no further adjustment for the obvious seasonality in dispersion: There is much less room for disagreement about "current" year earnings when, for instance, three of four quarters' results have already been reported. However, I did repeat the tests below after dividing dispersion by the square root of the length of time remaining to the end of the fiscal period. Results were unaffected.

Table I

Return Regressions

The table shows results from monthly Fama–MacBeth regressions of returns on measures of analyst forecast dispersion and leverage, and their product. The variable DISP1 is the standard deviation of current-fiscal-year forecasts divided by the mean of the forecasts. The variable DISP2 is the standard deviation divided by the firm's most recently reported asset value. Both measures are transformed into percentile rank form. The leverage measure L is the most recently reported book value of debt divided by the sum of that debt and the month-end market value of equity. The data are monthly observations from January 1983 through December 2001. The rightmost column is the arithmetic average of the R^2's of the individual regressions. The t-statistics are in parentheses.

Independent Variables	Dispersion	Leverage	Interaction	R^2
DISP1, L	−0.0059			0.42
	(2.46)			
	−0.0058	0.0049		1.44
	(2.68)	(1.24)		
	−0.0041	0.0071	−0.0044	1.53
	(1.44)	(2.23)	(1.19)	
		0.0103	−0.0113	1.33
		(2.19)	(3.28)	
DISP2, L	−0.0035			0.65
	(1.09)			
	−0.0027	0.0035		1.82
	(1.09)	(0.79)		
	0.0017	0.0089	−0.0124	2.00
	(0.60)	(2.32)	(2.27)	
		0.0074	−0.0097	1.80
		(1.45)	(2.02)	

diminishes.[13] I deal with this by transforming both dispersion measures into percentile ranks. Besides being more robust, this fits the data better, significantly increasing the R^2's in all cases. It also enables one to interpret the regression coefficients in terms of the performance of portfolios sorted by dispersion.

The basic results are shown in Table I. The top panel uses DISP1 and the bottom panel uses DISP2. With either measure, the main implications of the model are well supported. In the presence of the interaction term, there is no significant dispersion effect. The interaction coefficient itself is negative and significant (except in one specification). Finally, the leverage coefficient is positive and significant, as the model implies it should be.[14] In all, this is a striking collection of successes for the model. The interaction test especially—being purely motivated by theory, previously unexamined, and not obviously

[13] To see this another way, think of the model again as $a + b \cdot \text{leverage} \cdot (c - \text{dispersion})$. Fitting this with extremely large dispersion values requires taking c large to keep the leverage effect positive. But this forces b to be small to keep the leverage effect from being extreme for small levels of dispersion, where most of the observations are.

[14] All reported t-statistics are adjusted for heteroskedasticity and autocorrelation. The choice of lag length in estimating the covariance matrix of coefficients has no effect on any of the conclusions.

Table II

Return Regressions with Expected Return Controls

The table shows results from monthly Fama–MacBeth regressions of returns on measures of analyst forecast dispersion and leverage, and their product, as well as on controls for expected return of underlying assets. The variable SZ denotes log market value of equity; BM is log of the ratio of book equity to market value of equity; $R12$ is the total stock return over the 11 months preceding the previous month; and DISP1, DISP2, and L are described in the caption to Table I. The bottom two specifications employ versions of these variables that have been orthogonalized (via a pooled regression) with respect to the expected return controls. The data are monthly observations from January 1983 through December 2001. The rightmost column is the arithmetic average of the R^2's of the individual regressions. The t-statistics are in parentheses.

Independent Variables	Dispersion	Leverage	Interaction	R^2
DISP1, L,	−0.0004	0.0031	−0.0090	4.41
SZ, BM, $R12$	(0.13)	(0.85)	(2.36)	
DISP2, L,	0.0028	0.0034	−0.0131	4.46
SZ, BM, $R12$	(0.98)	(0.90)	(2.70)	
Orthogonalized DISP1, L,	−0.0038	0.0024	−0.0074	4.38
SZ, BM, $R12$	(1.82)	(0.68)	(2.00)	
Orthogonalized DISP2, L,	−0.0022	0.0036	−0.0124	4.39
SZ, BM, $R12$	(1.01)	(0.98)	(2.70)	

explainable by alternative theories—provides strong evidence in support of the basic insight behind the theory.

Examining the coefficients in terms of economic significance, the estimates imply that a low dispersion firm (10th percentile) with high leverage (90%) would outperform a high dispersion one (90th percentile) with low leverage (10%) by between 60 and 90 basis points per month. Firms with no debt have essentially the same expected return whether they are in the 99th dispersion percentile or the first. These numbers suggest that the model's consequences may be of practical as well as theoretical importance.

The specifications in Table I are unconditional however, and these initial conclusions could be sensitive to cross-sectional variation in the underlying firm parameters. Most important among these is the asset risk premium π (cf. equation (7)), which would characterize firm returns in the absence of debt. The model's predictions apply to the incremental effects of debt and dispersion with this fundamental expected return premium held fixed. I now try to control for possible cross-sectional differences in it. As the introduction noted, the theory here offers no new suggestions about how to identify the sources of fundamental risk that would drive these differences. So, in Table II, I simply introduce the standard controls: size, book-to-market ratio, and momentum.[15] The top two lines

[15] Using the book-to-market ratio as a proxy for fundamental risk in this context is a debatable call, because under the model, this ratio is also a function of leverage and parameter risk. The treatment here is traditional and facilitates comparison with the existing literature. However, it is worth pointing out that the model implies a positive role for book-to-market *even if it has no relation to systematic risk*. Holding systematic risk fixed, a lower ratio means higher idiosyncratic risk and hence—just as with dispersion—lower expected returns. Hecht (2002), in a cross-sectional

indicate that doing this does not affect the original findings. With either measure of dispersion, the interaction term is actually strengthened (more negative) and the dispersion term alone remains insignificant. The leverage coefficients are still positive but somewhat diminished. This is due to the strong association between leverage and book-to-market. The latter absorbs most of the explanatory power of the former, as noted by Fama and French (1992), and becomes highly significant in these specifications.

The next two rows of the table take the controls one step further by applying a two-step procedure that first orthogonalizes both leverage and dispersion with respect to the expected return proxies.[16] If leverage and dispersion are themselves endogenous quantities, influenced in part by the the cost of capital (risk premium) of the underlying business, then the estimated coefficients from the original regressions might still be distorted by the dual role the proxies are playing. The table shows, however, that this does not appear to be a concern. The estimated coefficients are little changed.

The other important parameter in the model not controlled for so far is the volatility of the underlying asset value, σ_V. Here there are also reasons for concern. As with the underlying risk premium, it is certainly plausible that this measure of risk covaries in the cross section with dispersion or leverage or both. And, again, endogeneity of leverage (and possibly of dispersion) clearly matters since riskiness of assets is undoubtedly a major determinant of the amount of debt firms take on. A new complication, also, is that leverage in turn influences stock volatility, which is the most tangible way of getting at asset risk. So, as a first step in controlling for variation in σ_V, I attempt to unlever the equity volatility estimated each month from CRSP returns. Then I follow the same steps as above: I include the control itself in the return regressions, and also orthogonalize the dispersion and leverage proxies with respect to it (along with all the earlier controls), and then use the residuals in place of the original variables.

Table III shows the results.[17] The first two regressions employ a crude deleveraging that sets $\hat{\sigma_V} = \hat{\sigma_P}(1 - L)$. The bottom two regressions employ the model's own dynamic relation between these two volatilities to solve for σ_V. This procedure requires simultaneously solving for the model's leverage ratio, $M \equiv e^{-r\tau}K/S$, which I interpret as the face value of debt over market value of assets.[18] So for consistency I use this measure in the regressions as well.

Either way, the previous findings are upheld. There is no significant dispersion effect independent of leverage, and the interaction is significant and

study of total firm returns that effectively strips out the influence of leverage, finds that most of the book-to-market effect does in fact vanish, and concludes that the ratio's apparent influence on returns must stem from capital structure effects. The model here provides a formal explanation for exactly how these capital structure effects might work.

[16] This is carried out by a single pooled regression.

[17] Here I only show the results for DISP2 for brevity. The findings are the same with DISP1.

[18] To solve for the volatility and market value of the firm, the model requires a time horizon, T, and a risk-free rate r. I used 4 years and 4% for these, but the results are not sensitive to these choices. Details are available from the author upon request.

Table III

Return Regressions with Expected Return and Volatility Controls

The table shows results from monthly Fama–MacBeth regressions of returns on measures of analyst forecast dispersion and leverage and their product, as well as on controls for expected return and volatility of underlying assets. The variable $\widehat{\sigma_P}$ is the prior month standard deviation of daily stock returns; $\widehat{\sigma_V}$ and M are the implicit model values of asset volatility and leverage, respectively, (the latter defined as present value of debt over firm market value) that simultaneously match the observed market value of equity and equity return volatility. The orthogonalized variables are residuals from a first-stage pooled regression on the controls. The variables SZ, BM, and $R12$ are described in the caption to Table II. The variables DISP1, DISP2, and L are described in the caption to Table I. The data are monthly observations from January 1983 through December 2001. The rightmost column is the arithmetic average of the R^2's of the individual regressions. The t-statistics are in parentheses.

Independent Variables	Dispersion	Leverage	Interaction	R^2
DISP2, L,	0.0038	−0.0039	−0.0143	5.28
$SZ, BM, R12, \widehat{\sigma_P}(1-L)$	(1.61)	(1.24)	(2.96)	
Orthogonalized DISP2, L,	−0.0017	−0.0018	−0.0175	5.22
$SZ, BM, R12, \widehat{\sigma_P}(1-L)$	(0.38)	(0.51)	(3.00)	
DISP2, M,	0.0029	−0.0008	−0.0100	5.32
$SZ, BM, R12, \widehat{\sigma_V}$	(1.29)	(0.35)	(2.70)	
Orthogonalized DISP2, M,	−0.0013	−0.0017	−0.0082	5.28
$SZ, BM, R12, \widehat{\sigma_V}$	(0.70)	(0.81)	(2.26)	

negative. These specifications show no separate role for leverage, which, as above, is not inconsistent with the model. The book-to-market ratio is picking up the role of debt while also proxying for the positive role of other model parameters.[19] It seems safe to conclude, at this point, that the initial success of the theoretical predictions was not due to any confounding cross-sectional correlation between the main proxies and other factors influencing equity's risk and return.

To summarize, this section has documented a new significant feature of the cross section of expected returns: a negative interaction effect between leverage and dispersion of beliefs. Rather than presenting a new puzzle, however, this pattern was predicted by a simple classical model of information and risky debt. That model, in turn, accounts for the original puzzle of a negative effect of dispersion alone. The results serve as a reminder of the complex and important role played by capital structure in continuously altering the exposure of equity to the underlying risks of a firm's business.

IV. Concluding Remarks

One of the most intriguing aspects of the original phenomenon reported by Diether, Malloy, and Scherbina (2002) is the suggestion that, to the extent that

[19] Another interpretation of the role of B/M, suggested by Brennan, Wang, and Xia (2002), is as a control for the duration of cash-flows, which is T here. As such, it would also be expected to enter positively: Longer duration (lower B/M) means higher option value and lower expected returns (see footnote 15 above).

dispersion of earnings expectations is under the control of firms themselves, they might actually benefit, via a lower cost of equity capital, by *increasing* disagreement. Or, at the very least, the finding casts strong doubt on the notion that transparent disclosures and timely earnings guidance are in a firm's own best interests. The results of this paper not only support these impressions, but also offer a straightforward explanation. For levered firms, adding idiosyncratic uncertainty about cash flows increases the option value of equity. In fact, the situation is exactly analogous to the so-called asset substitution agency problem in corporate finance, except that here a separate—and much simpler—channel is involved. No assets need be substituted to achieve the desired result. All that is required is obfuscation.

Does this happen? In a sense, the thesis of this paper is that it should, at least when firms have debt. And this prediction is also supported in additional regressions reported by DMS. High (book) leverage and poor past performance are associated with higher dispersion in the cross section, suggesting that firms with higher incentives to dissemble may in fact do so. This interpretation is bolstered by the surprising fact that high book-to-market ratios also entail higher dispersion. One might have expected the reverse: that dispersion might be higher for firms with more intangible value. Instead distressed firms are more prone to analyst disagreement despite their smaller intangible component. It seems plausible that this is due, at least in part, to a deliberate response of managers to bad news. The model here offers a rationale for such behavior.

Understanding the complete game determining the amount and accuracy of information reported by managers is a topic of enormous current interest and importance. This paper has only attempted to model the asset pricing consequences of an exogenous information setting. Still, it demonstrates that this is a crucial facet of the problem that yields some striking implications. Moreover, there appears to be compelling empirical support for its basic linkage between leverage and uncertainty about fundamentals in determining expected returns.

Appendix: Filtering Equations

This appendix provides explicit representations for the endogenous quantities that arise in the filtering problem of Section II.A.

To start, write the complete system of state, v_t, and observation, $(u_t, \lambda_t) \equiv (\log U_t, \log \Lambda_t)$, evolution as

$$
\begin{aligned}
dv_t &= \bar{\epsilon}\, dt + \sigma_V d W_t^V,\\
du_t &= \bar{\epsilon} - \kappa(u_t - v_t)\, dt + \sigma_U d W_t^U,\\
d\lambda &= -\bar{r}\, dt + \sigma_\Lambda d W_t^\Lambda,
\end{aligned}
$$

where $\bar{r} \equiv (r + (1/2)\sigma_\Lambda^2)$ and $\sigma_U\, dW^U \equiv \sigma_V\, dW^V + \sigma_\eta\, dW^\eta$. Notice that the second line uses $\eta = u - v$ to eliminate any explicit reference to the noise process.

Let Σ be the covariance matrix between the two observable processes. That is,

$$\Sigma = \begin{bmatrix} \sigma_U^2 & \sigma_{U\Lambda} \\ \sigma_{U\Lambda} & \sigma_\Lambda^2 \end{bmatrix}.$$

When the error component of the signal, η, is assumed to be orthogonal to both V and Λ, then $\sigma_{U\Lambda}$ is also equal to $\rho_{V\Lambda}\sigma_V\sigma_\Lambda$. Let $\Sigma^{-1/2}$ denote any matrix square root of Σ.

Now define the bivariate innovation process $d\widehat{W}_t$ to be

$$d\widehat{W}_t = \Sigma^{-1/2} \begin{bmatrix} du_t - \tilde{E}_t[du_t] \\ d\lambda_t - \tilde{E}_t[d\lambda_t] \end{bmatrix}.$$

Then the results of Section 12.3 in Liptser and Shiryaev (1977) imply that $d\widehat{W}_t$ is a standard Brownian motion, and that the conditional expectation $\tilde{m}_t \equiv \tilde{E}_t[v_t]$ evolves according to

$$d\tilde{m}_t = \bar{\epsilon}\,dt + \hat{h}_t'\,d\widehat{W}_t,$$

where $\hat{h}_t$ is given by

$$\hat{h}_t = \Sigma^{-1/2} \begin{bmatrix} \sigma_{VU} + \kappa\tilde{\omega}_t \\ \sigma_{V\Lambda} \end{bmatrix}.$$

This vector gives the sensitivity of the conditional expectation to the two sources of information. The terms σ_{VU} and $\sigma_{V\Lambda}$ reflect the known covariances of the unobservable process with the two types of innovation. The additional term $\kappa\tilde{\omega}_t$ increases the reaction of $\tilde{m}$ to u shocks because v appears in the true drift of u. Here the intuition is more subtle. Even if shocks to u were uncorrelated with shocks to v, the presence of u in the drift means that some part of the observed surprise in u (i.e., $du_t - \tilde{E}_t[du_t]$) will have been due to misestimation of the true drift. Bayesian updating will then try to improve estimates of the level of v by exploiting this extra information. Positive surprises probably mean that the drift was underestimated, and so $\tilde{m}$ should be increased based on the portion of the surprise that was likely due to this error.

To connect the above vector quantities to the notation used in the text, note that the scalar processes $\tilde{h}_t$ and $\widetilde{W}_t$ can be defined implicitly by the relations

$$\tilde{h}_t\,d\widetilde{W}_t = \hat{h}_t'\,d\widehat{W}_t$$

and

$$\tilde{h}_t^2 = \hat{h}_t'\hat{h}_t.$$

Next, the classical analysis also yields the result that the conditional variance process $\tilde{\omega}_t$ solves the ordinary differential equation

$$\sigma_V^2 + \frac{d\tilde{\omega}_t}{dt} = \hat{h}_t'\hat{h}_t = \tilde{h}^2(\tilde{\omega}_t).$$

The steady-state value must solve the time-homogeneous version of this, which shows that $\tilde{h}^2(\tilde{\omega}) = \sigma_V^2$, as claimed in the text. Solving explicitly for the limiting value $\tilde{\omega}$ yields

$$\tilde{\omega} = \frac{\sigma_V^2}{\kappa}\left(1 - \rho_{V\Lambda}^2\right)\left\{\sqrt{1 + \frac{\left(1 - \rho_{VU}^2\right)}{\left(1 - \rho_{V\Lambda}^2\right)}} - 1\right\}.$$

This is the expression plotted in Figure 2. It does use the orthogonality assumptions on η, which also imply $\rho_{\eta V}^2 = (\sigma_\eta/\sigma_V)^2 = (1 - \rho_{VU}^2)$ (see also footnote 5).

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